Video – 5

Review

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What we did in this course

We went through the following topics in this course:

Basic Principles

- Lesson 1: Multiplication and addition principles
- Video 1: Inclusion-exclusion principle
- Video 3: Overcounting
- Lesson 5: Probability basics

Permutations and Combinations

- Lesson 2: Permutations
- Lesson 3: Combinations
- Lesson 4: Repetitions (repeated permutations)
- Video 4: Stars and bars (repeated combinations)

Basic Principles

Multiplication Principle

You make a sequence of decisions to reach the end goal.

Each decision does not affect the number of ways for next decisions.

Then, we multiply the number of ways for each decisions.

Addition Principle and Inclusion-Exclusion Principle

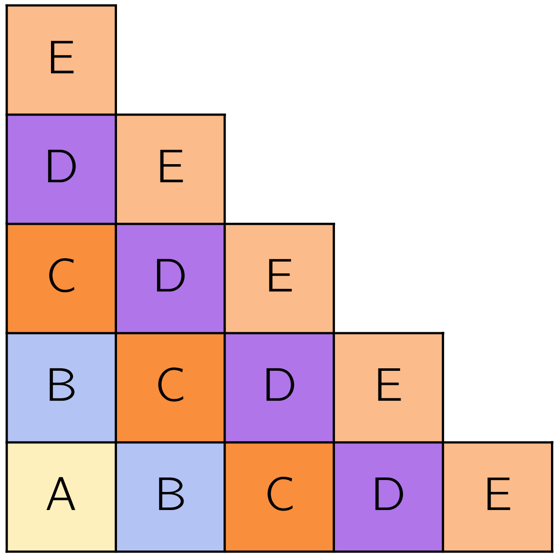
You split what you want to count into multiple types. If the types are non-intersecting, just add each type. If they are intersecting, add them and subtract their intersection (for 2 types).

Overcounting

Do not care about repetitions while counting. Divide by the number of times repeated.

Q1. Spelling ABCDE

Andy the little ant starts at square A in the figure. He then walks around the grid to spell the word ABCDE by passing through the squares with B, C, D and E written on them in that order. In how many ways can Andy finish his spelling?

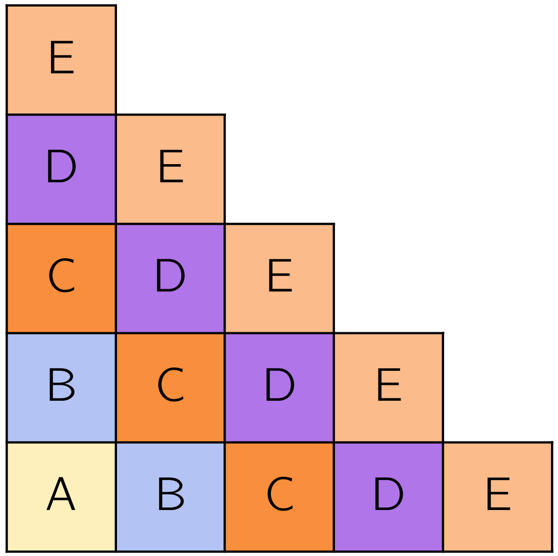


Q1. Spelling ABCDE

Solution

Andy has to make 4 decisions:

- Decision 1: Choose which B to walk to.
- Decision 2: Choose which C to walk to.
- Decision 3: Choose which D to walk to.
- Decision 4: Choose which E to walk to.

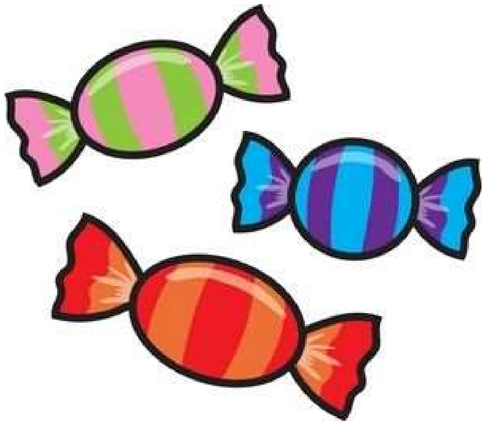


Each decision can be made in two ways (go up or go right).

So, number of ways is $2 \times 2 \times 2 \times 2 = 16$.

Q2. Sweet Start or Sweet End

Thandar has six different types of candies of which orange and strawberry are her favourite. She wants to eat four candies, one after another. But she wants to either start or end (but not both) with her favourite candy. In how many ways can she eat her candies?

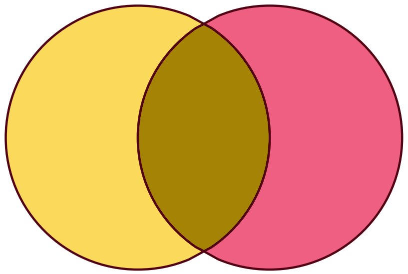


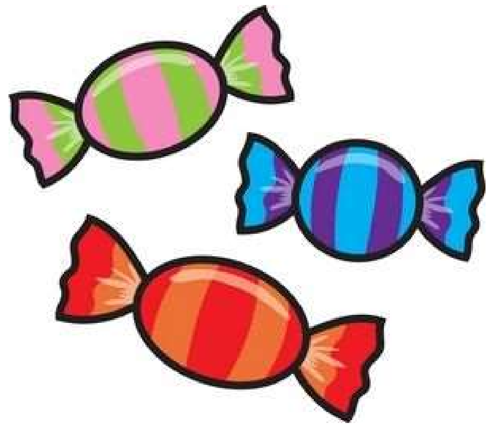
Q2. Sweet Start or Sweet End

Solution

We can group the eating sequences into two (intersecting) types:

- Type 1: Sequences starting with her favourite,
- Type 2: Sequences ending with favourite.





Number of type $1 = 2 \times 5 \times 4 \times 3 = 120$.

Number of type $2 = 2 \times 5 \times 4 \times 3 = 120$.

Number of sequences in intersection $= 2 \times 4 \times 3 = 24$.

Therefore, number of ways is $120 + 120 - 2 \times 24 = 192$.

Q3. Monkey Experiment

To study the behaviour of monkeys, scientists brought 18 monkeys to the lab. The monkeys are divided into 6 groups with 3 monkeys in each group. Monkeys in the same group then gets to know each other.

Next, the scientists pick two monkeys in different groups and let them introduce each other. How many times do the scientists need to do this so that all monkeys know each other?

Q3. Monkey Experiment

Solution 1 (using combinations)

Ways to pick 2 monkeys out of 18 = $C_2^{18} = 153$.

Number of pairs within the same group = $6 \times C_2^3 = 18$.

Therefore, number of introductions $= 153 - 18 = 135$.

Solution 2 (using overcounting)

Each monkey needs to make $18 - 3 = 15$ introductions.

But, 15×18 overcounts each introduction twice.

Therefore, number of introductions is $\dfrac{15 \times 18}{2} = 135$.

Permutations and Combinations

Permutations with no repetitions

Number of ways to arrange r objects out of n different objects is P_r^n.

Relavant formulae

$$P_r^n = n \times (n-1) \times (n-2) \times \cdots \times (n-(r-1)) = \frac{n!}{(n-r)!}$$

Combinations with no repetitions

Number of ways to select r objects out of n different objects is C_r^n.

Relevant formulae

$$C_r^n = \frac{n \times (n-1) \times (n-2) \times \cdots \times (n-(r-1))}{r!} = \frac{n!}{r! \times (n-r)!}.$$

Repeated permutations

Given n objects in which a are of one type, b are of another type and so on. Then,

number of ways to arrange all n objects is $\dfrac{n!}{a! \times b! \times \cdots}$.

Q4. All Permutation Tricks

In how many ways does the letters of the word TRICKS be arranged so that

(a) it starts with S?

(b) the letters C, K and S are together?

(c) the letters T, R and I are never adjacent to each other?

Q4. All Permutation Tricks

(a) it starts with S?

We just need to permute TRICK. Number of ways $= 5! = 120$.

(b) the letters C, K and S are together?

Put C, K and S in a box. Put others in their own box.

[T][R][I][CKS]

We permute the boxes, then inside each box.

So, number of ways $= 4! \times 3! = 24 \times 6 = 144$.

(c) the letters T, R and I are never adjacent to each other?

Arrange CKS first. This creates 4 gaps: _ K _ S _ C _.

Put T, R, I in these gaps. So, number of ways $= 3! \times 4 \times 3 \times 2 = 144$.

Q5. All Permutation Tricks (repeated)

Nyo Kharr wants to arrange the letters of the word CHOCOPOT. In how many ways can she do this if

(a) there are no restrictions?

(b) the letter O's must stay together?

(c) the two C's need to be separated?

Q5. All Permutation Tricks (repeated)

(a) there are no restrictions?

Number of ways = $\dfrac{8!}{2! \times 3!} = 3360.$

(b) the letter O's must stay together?

Put O's in one box, others in their own boxes:

[C][H][C][P][T][OOO].

Number of ways to arrange the boxes $= \dfrac{6!}{2!} = 360.$

(c) the two C's need to be separated?

Arrange HOOPOT first. This creates 7 gaps for C's: _H_O_O_P_O_T_.

Place two C's in them. So, number of ways = $\dfrac{6!}{3!} \times C^7_2 = 2520.$

Q6. Flowers for Everyone

Khayay Phyu plucked nine different flowers including a rose and an orchid. She wants to give three of these flowers to Hnin Si and three to Thit Khwa. In how many ways can she give if

(a) there are no restrictions?

(b) she wants to give a rose to Hnin Si and an orchid to Thit Khwa?

(c) she wants to keep the rose and orchid for herself?

Q6. Flowers for Everyone

(a) there are no restrictions?

We choose 3 flowers for Hnin Si, then 3 for Thit Khwa.

So, number of ways is $C_3^9 \times C_3^6 = 1680.$

(b) she wants to give a rose to Hnin Si and an orchid to Thit Khwa?

We give the rose and orchid to them first. Then, we choose 2 flowers for Hnin Si, then 2 for Thit Khwa. So, number of ways $= C_2^7 \times C_2^5 = 210.$

(c) she wants to keep the rose and orchid for herself?

Remove the rose and orchid. Then, we choose 3 flowers for Hnin Si, then 3 for Thit Khwa. So, number of ways $= C_3^7 \times C_3^4 = 140.$

Q7. Permute and Combute

There are two rows of seats available, each with 5 seats in them. Sayarma wants to call 3 boys and 3 girls from her classroom of 10 boys and 8 girls. She wants the students to sit so that all boys sit in one row and all girls sit in the other. In how many ways can Sayarma choose and seat her students?

Q7. Permute and Combute

Solution

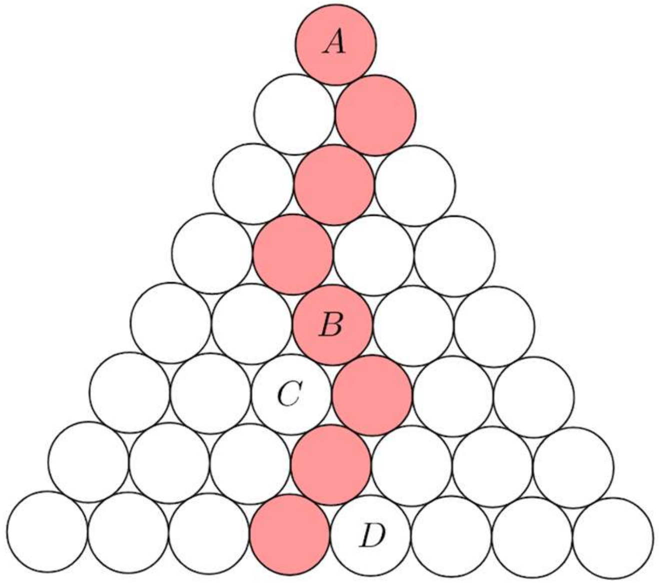
Sayarma has to do following decisions:

- Select 3 boys out of 10.
- Select 3 girls out of 8.
- Choose whether the boys sit in the front or girls sit in the front.
- Permute the chosen boys in the 5 seats.
- Permute the chosen girls in the 5 seats.

Therefore, number of ways is $C_3^{10} \times C_3^8 \times 2 \times P_3^5 \times P_3^5 = 120 \times 56 \times 2 \times 60 \times 60 = 48384000.$

Q8. Ninja Paths

The figure shows 36 circles arranged in an triangular array. Nay Htoo Naing the ninja starts in the circle at the top row, and he wants to go to one of the circles at the bottom row. In each move, he can move down to either of the two circles touching the circle he is in. For example, one of the paths he could take is shown in red.



(a) He wants to end his trail at circle D. In how many ways can he do this?

(b) He wants his trail to include circle B and circle D. In how many ways can he plan his trail?

Q8. Ninja Paths

Solution

(a) He wants to end his trail at circle D. In how many ways can he do this?

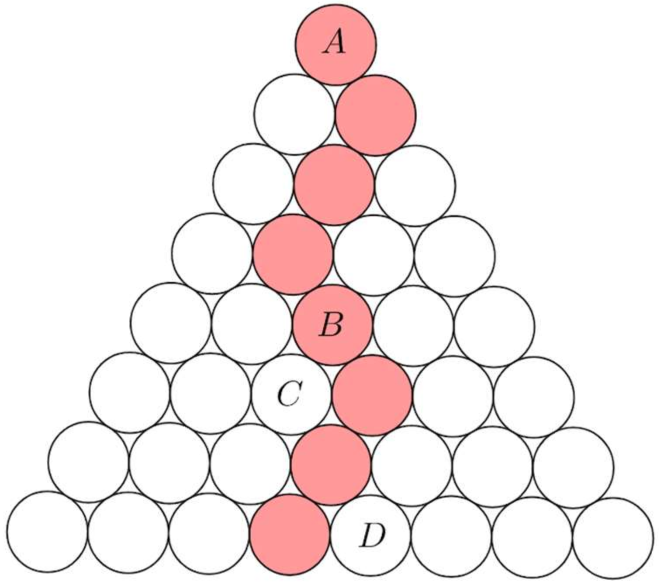
To get from A to D, we have to go left 3 times and right 4 times.

So, paths from A to D can be represented as permutations of LLLRRRR. Hence, number of ways $= \dfrac{7!}{3! \times 4!} = 35.$

(b) He wants his trail to include circle B and circle D. In how many ways can he plan his trail?

He need to go from A to B, then from B to D.

Hence, number of ways $= \dfrac{4!}{2! \times 2!} \times \dfrac{3!}{2!} = 18.$



Topics not covered here

I skipped over the following topics in this video:

- Stars and bars
- Probability
- Combinatorial identities

As for the exam, there will be one question on stars&bars and a few on simple probability computations. I will not ask anything about combinatorial identities.

That's it for this video.

Good luck with the test!

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