Video – 3

# Overcounting and Circular Permutations

COM

I

# Section – I

## Overcounting

COM

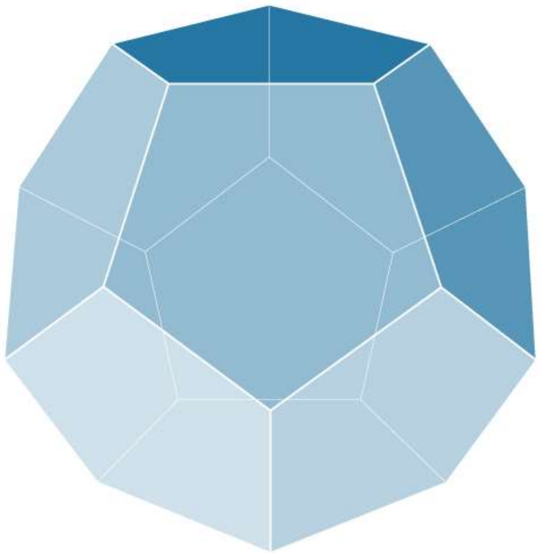
I

# Q1. Vertices and Edges of Dodecahedron

It is known that the solid shown in picture is made up using 12 pentagons with three pentagons meeting at each vertex. How many vertices and edges does this solid have?

Vertices: 20

Edges: 30



Solution

There are 12 pentagons with 5 vertices on each of them.

But, $12 \times 5 = 60$ counts each vertex three times.

Thus, number of vertices is $60/3 = 20$.

Similar reasoning works for edges. Number of edges $= \dfrac{12 \times 5}{2} = 30$

# Q2. Vertices and Edges of a Soccer Ball

It is known that the surface of a standard soccer ball is made up with 12 pentagons and 20 hexagons. Three faces meet at each vertex.

How many vertices and edges does a standard soccer ball have?

Vertices: 60

Edges: 90

Solution

$$\text{Number of vertices} = \frac{12 \times 5 + 20 \times 6}{3} = 60.$$

$$\text{Number of edges} = \frac{12 \times 5 + 20 \times 6}{2} = 90.$$

# Q3. Groups in States

In a state, there are 15 groups of people, each of which has exactly 100 people in it. If every person is in exactly 4 of the groups, find the number of people in the state.

Answer: 375

Solution

Each group has 100 people and there are 15 groups.

But, $100 \times 15$ counts each person 4 times.

Therefore, the answer is $\dfrac{100 \times 15}{4} = 375.$

## The Idea of Overcounting

General Strategy:

- First, count what you want to count without caring about repetitons.
- Then, divide your answer by the number of repetitions each object had.

# Q4. Counting Handshakes

In a party, whenever a guest arrives, he shakes hands once with every guest in the party that is a friend of him. When all 20 guests arrived, it is found that everyone had shaken their hands exactly 5 times. How many handshakes are there?

Answer: 50

Solution

There are 20 people and each person shakes 5 times.

But, $5 \times 20 = 100$ counts each shake twice.

Therefore, the answer is $50$.

# Q5. Choosing 4 Letters

Sai Sint has 7 letters A, B, C, D, E, F, G. He wants to choose four of these letters. In how many ways can he choose?

Solution

Overcount by letting Sai Sint make 4-letter words.

Then, there are $7 \times 6 \times 5 \times 4$ such 4-letter words.

But then, each choice is overcounted $4 \times 3 \times 2 \times 1 = 24$ times.


Therefore, number of choices is $\dfrac{7 \times 6 \times 5 \times 4}{24} = 35.$

## Section – II

## Circular Permutations

COM

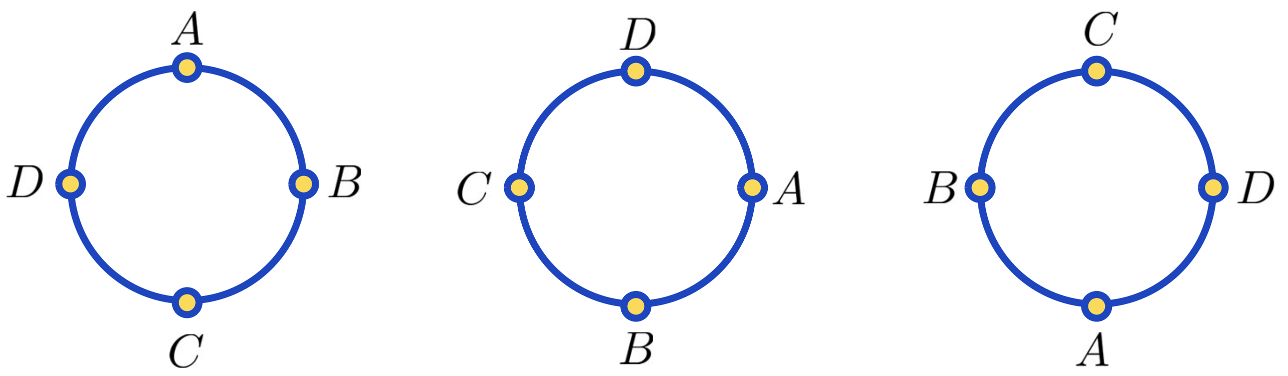
I

# What it means to arrange in a circle

Question: What's the difference between arranging A, B, C, D in a row, and around a circle?

Answer: When arranging in a circle, we usually define the arrangements to be "equal" if we can obtained from the other by a rotation.

For example, as row permutations ABCD, BCDA, CDAB are different. But, as a permutation around a circle, they are the same.



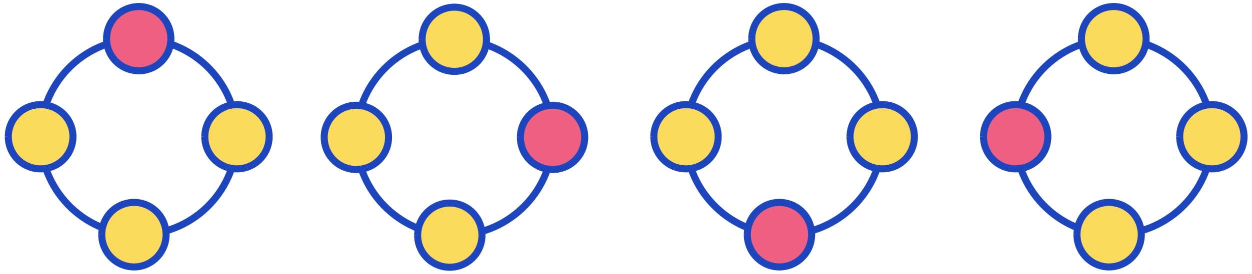
# Q6. Our First Circle

In how many ways can we place Aye Aye, Bo Bo, Chaw Chaw and Dar Dar around a round-table? Answer: 6

Solution

There are $4 \times 3 \times 2 \times 1 = 24$ ways to arrange them in a circle if rotation is not considered.

Now, let's think about how many times we overcount each circle.



We overcounted each circle four times.

Therefore, number of circular arrangements is $6$.

# Circular Permutations Formula

Number of ways to arrange $r$ objects out of $n$ different objects in a circle is equal to

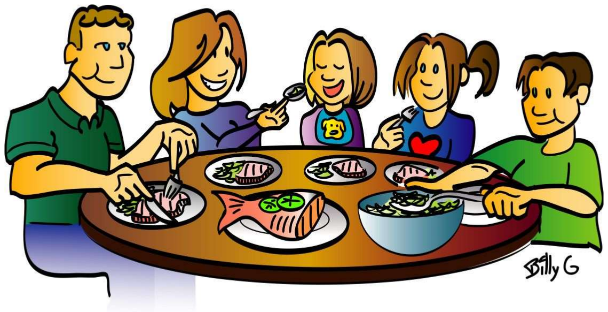
$$\frac{P_r^n}{r} = \frac{n \times (n-1) \times (n-2) \times \cdots}{r}$$

Reason

- There are $P_r^n$ ways to put them around the circle if we do not care about rotations.
- But, this overcounts each circle $r$ times when we consider rotations.
- Therefore, number of circular arrangments is $P_r^n$ divided by $r$.

# Q7. Aye Family's Dinner

Aye Aye, Aye Mi and Aye Chan are children of U Chan Aye and Daw Mi Mi Aye. They want to sit around a round-table for family dinner.



(a) In how many ways can they sit? Answer: 24

(b) In how many ways can they sit if the parents must sit together?

Answer: 12

# Q7. Aye Family's Dinner

Solution

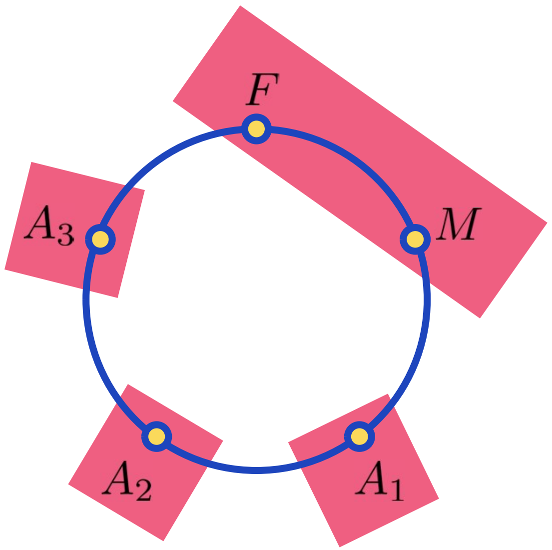
(a) By circular permutation formula, the answer is

$$\frac{P_5^5}{5} = \frac{5 \times 4 \times 3 \times 2 \times 1}{5} = 24.$$

(b) Put people in boxes as usual. We need to make 2 decisions

- Arrange the boxes in a circle. ← Circular
- Arrange the parent's in their box. ← NOT circular

Number of ways is $\frac{P_4^4}{4} \times P_2^2 = \frac{4 \times 3 \times 2 \times 1}{4} \times 2 \times 1 = 12.$



# Q8. Model Round-up

Khin San wants to invite 10 male models out of 15 and 5 female models out of 5, then seat them in a 15-seat round-table for a treat. Suppose Khin San wants to keep female models separated. In how many ways can she do this?

Answer: 21794572280

Solution

Same old separation trick. We make two decisions.

- Decision 1: Place male models in a circle. ← Circular
- Decision 2: Place female models in the 10 gaps formed. ← NOT circular

Therefore, the number of ways is $\dfrac{P^{15}_{10}}{10} \times P^{10}_{5}$

That's it for this video.

See you next lesson!

COM

I