Video – 2

Angle Bashing

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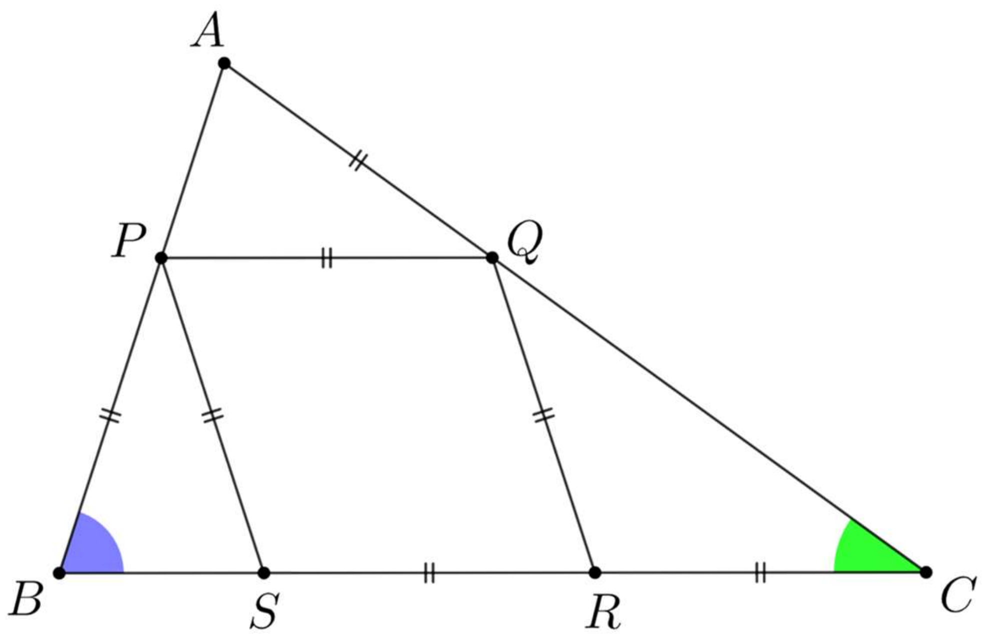
Q1. Rhombus inside Triangle

In the figure, PQ = QR = RS = PS = AQ = BP = CR. Find ∠B and ∠C.



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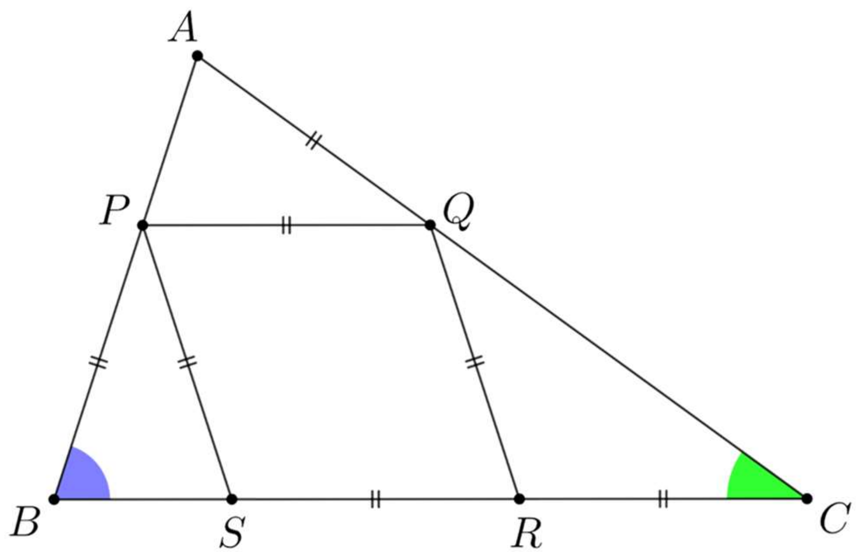


Solution

Let x = blue and y = green. Then,

By moving angles around, x = 2y

PQRS is rhoumbus and so PQ || BC.

So, ∠A = x as well.

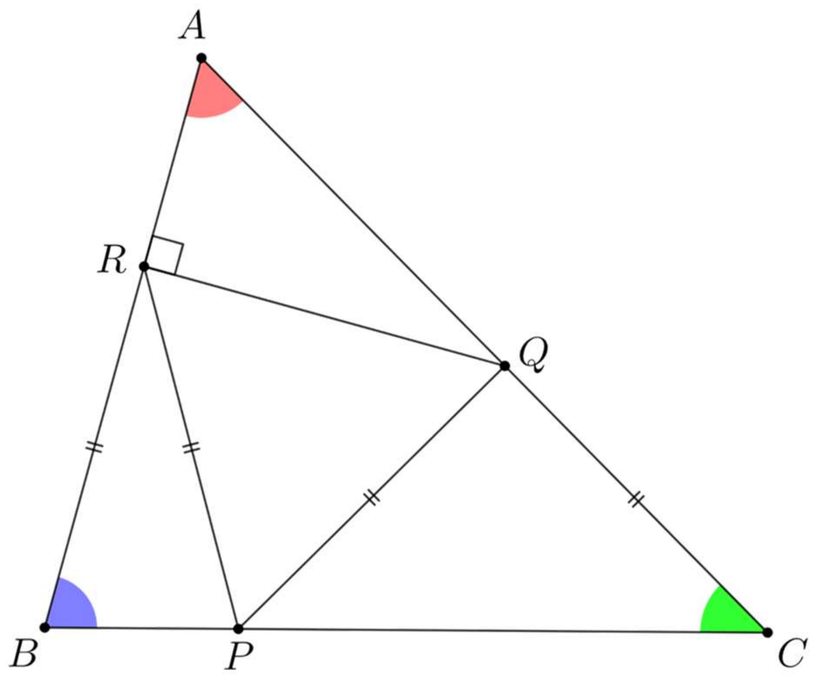
Hence,

x + x + y = 180° gives 5y = 180°.

Therefore, y = 36° and x = 72°.

Q2. Too Many Conditions for a Good Name

In triangle ABC, $\angle A = 60°$, $BR = RP = PQ = QC$ and $QR \perp AB$. Find $\angle B$ and $\angle C$.



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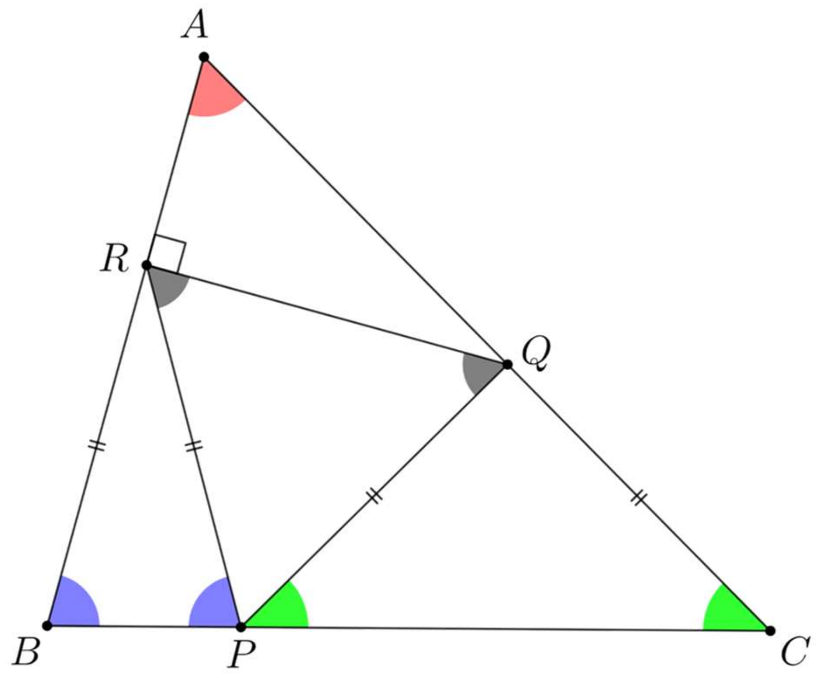


Solution

Let x = blue and y = green.

Then, $x + y = 120°$ ....(eq1)

Note that $\angle AQR = 30°$.

Looking at PQC, $2y = \text{grey} + 30°$.

Looking at BPR, $2x = \text{grey} + 90°$.

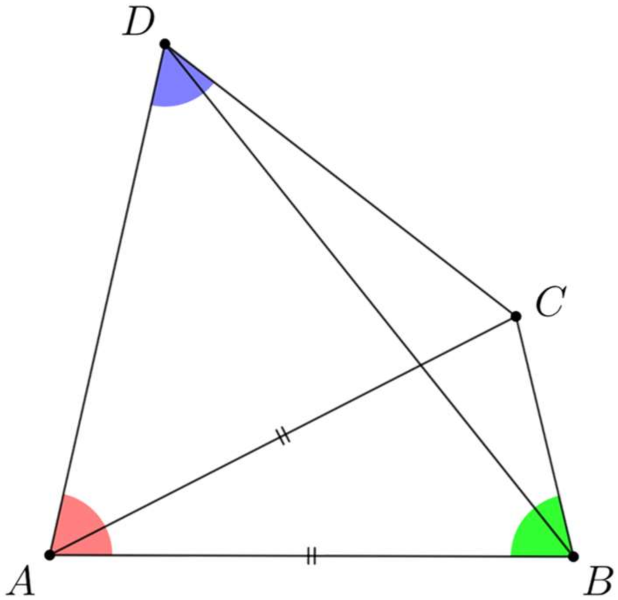
Hence, $2x - 2y = 60°$ i.e. $x - y = 30°$ ....(eq2)

Solving (eq1) and (eq2), we get

$x = 75°$ and $y = 45°$.

Q3. Equal Lengths in a Quadrilateral

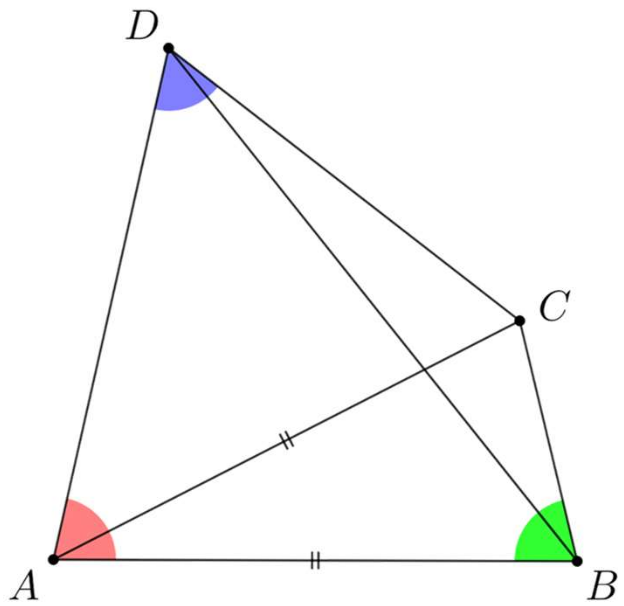
In quadrilateral ABCD, AB = AC, $\angle DAB = 80°$, $\angle ABC = 75°$ and $\angle CDA = 65°$. Find $\angle CDB$.



Q3. Equal Lengths in a Quadrilateral

In quadrilateral ABCD, AB = AC, $\angle DAB = 80°$, $\angle ABC = 75°$ and $\angle CDA = 65°$. Find $\angle CDB$.

Solution



$\angle DCA = 360° - (80° + 75° + 75° + 65°) = 65°$.

Therefore, AC = AD.

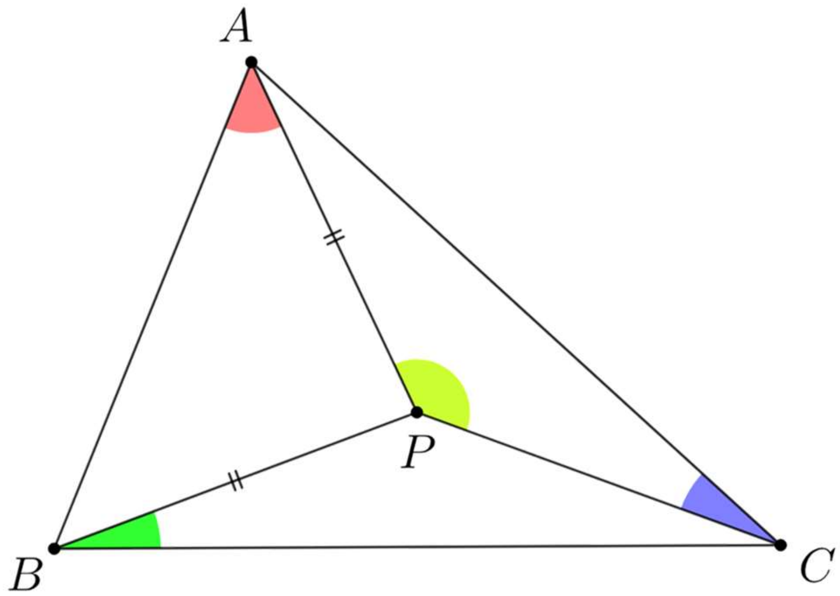
So, AB = AD.

This gives $\angle ADB = 50°$.

Therefore, $\angle CDB = 65° - 50° = 15°$.

Q4. Fan-shaped Angles

Let P be a point inside an acute triangle ABC such that PA = PB. Suppose that $\angle PAB = 40°$, $\angle PBC = 20°$ and $\angle APC = 120°$. Find $\angle ACP$.



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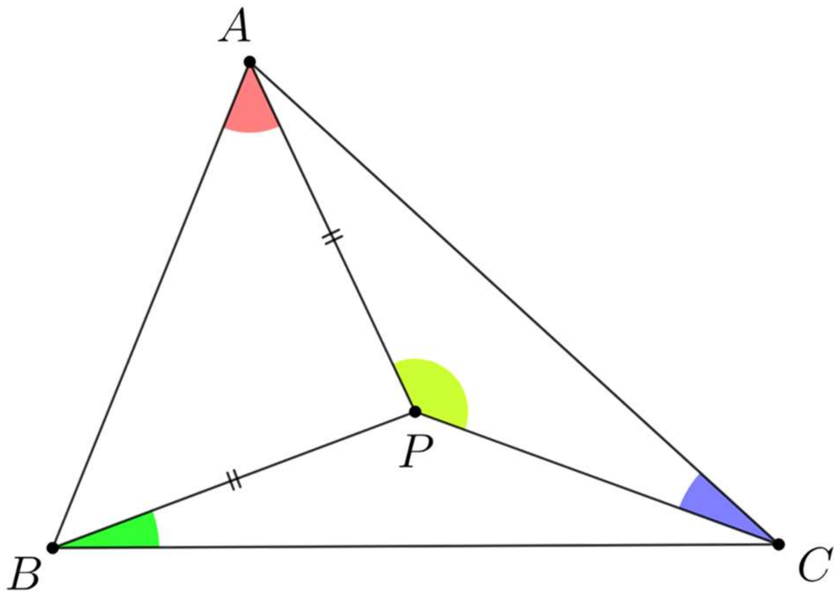


Solution

Note that $\angle APB = 180° - (40° + 40°) = 100°$.

Therefore, $\angle BPC = 360° - (100° + 120°) = 140°$.

Hence, $\angle PCB = 180° - (140° + 20°) = 20°$.

Therefore, $\angle PBC = \angle PCB$.

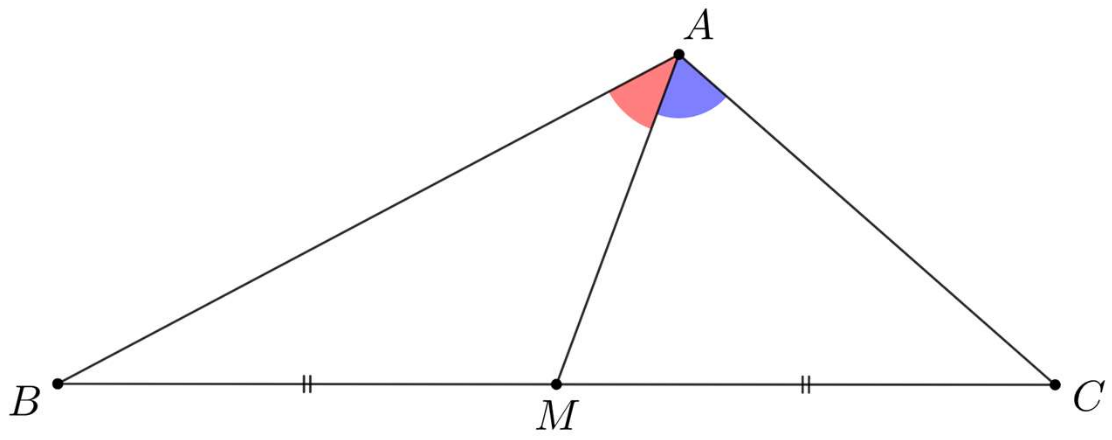
So, PB = PC and hence PA = PC.

Thus, $\angle ACP = (180° - 120°)/2 = 30°$.

Q5. Twice the Median

In triangle ABC, M is the midpoint of side BC. Suppose that ∠BAM = 30° and ∠MAC = 75°.

Prove that AB = 2AM.

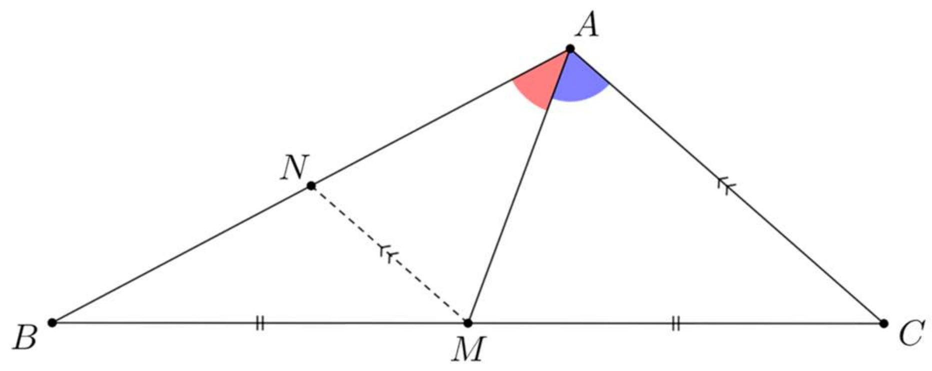


Q5. Twice the Median

In triangle ABC, M is the midpoint of side BC. Suppose that $\angle BAM = 30°$ and $\angle MAC = 75°$.

Prove that $AB = 2AM$.

Solution



Let N be midpoint of AB.

Our goal is to show that $AN = AM$.

Note that $MN \parallel AC$.

Thus, $\angle AMN = \text{blue} = 75°$.

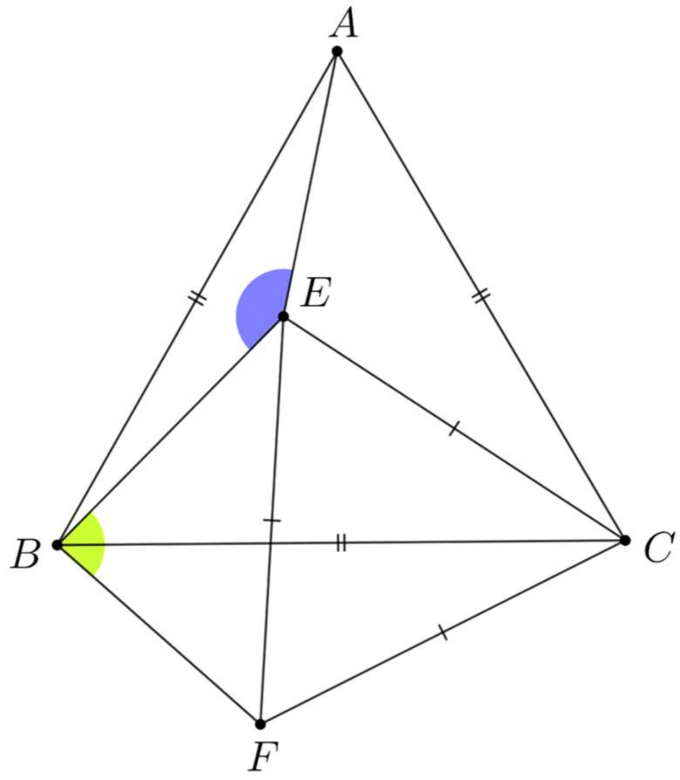
And $\angle ANM = 180° - (30° + 75°) = 75°$.

Therefore, $\angle AMN = \angle ANM$.

Hence, $AN = AM$.

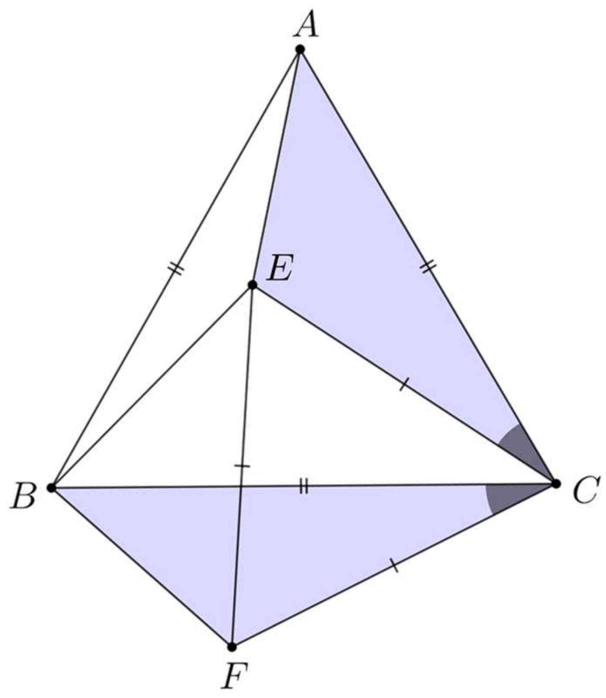
Q6. Rotated Equilaterals

In the figure, ABC and CEF are equilateral triangles. Suppose that ∠FBE = 85°. Find ∠AEB.



Q6. Rotated Equilaterals

In the figure, ABC and CEF are equilateral triangles. Suppose that $\angle FBE = 85°$. Find $\angle AEB$.



Solution

Note that $\angle ACE = 60° - \angle BCE$.

Also, $\angle BCF = 60° - \angle BCE$.

Therefore, $\angle ACE = \angle BCF$.

Hence, ΔCAE and ΔCBF are congruent by SAS.

Now, $\angle BEF + \angle BFE = 95°$.

So, $\angle BEC + \angle BFC = 215°$.

So, $\angle BEC + \angle CEA = 215°$.

Therefore, $\angle BEA = 360° - 215° = 145°$.

That's it for this video.

See you soon!

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