Video – 1

Extra Area Hunting

GEO

I

Section – I

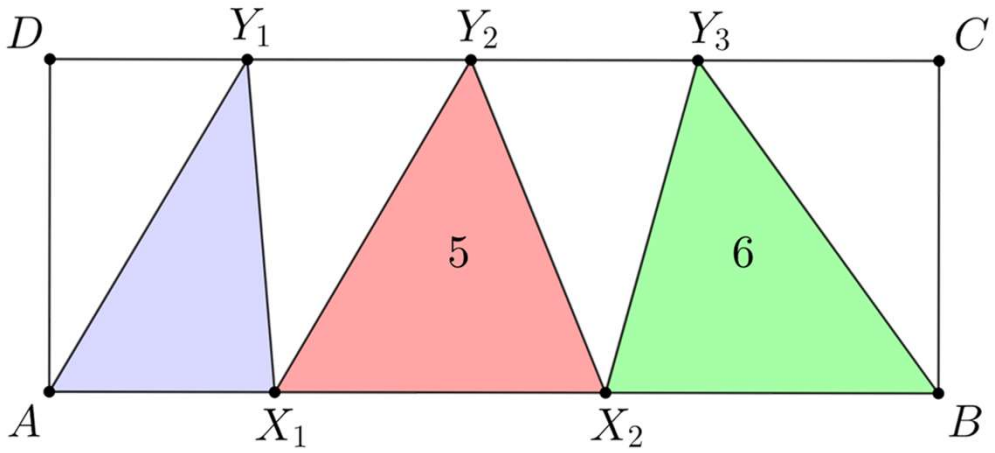
Sliding Triangles

GEO

I

Q1. Triple Spikes

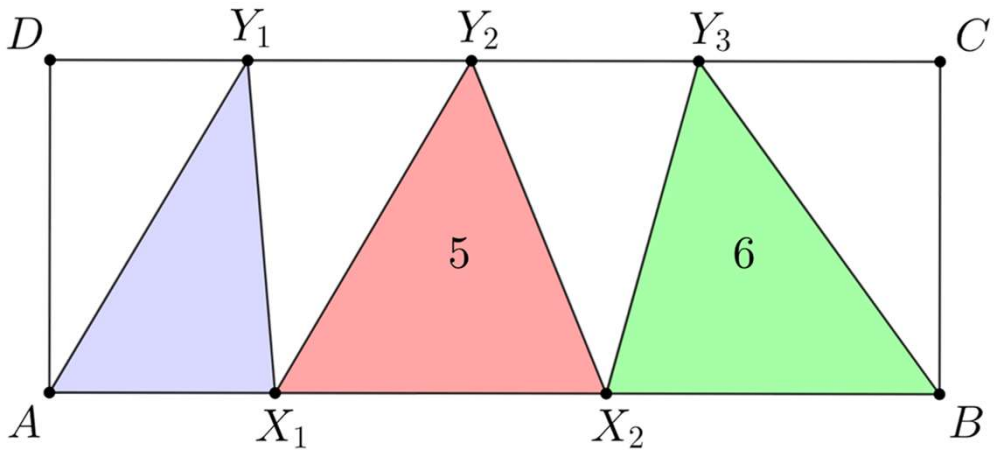
In the picture, ABCD is a rectangle with area 30. Area of the red triangle is 5 and the green triangle is 6. What is the area of the blue triangle?



Q1. Triple Spikes

Computational Solution

Let area of AX_1Y_1 be x.



By given,

- $AB \times BC = 30$,
- $AX_1 \times BC = 2x$,
- $X_1X_2 \times BC = 10$,
- $X_2B \times BC = 12$.

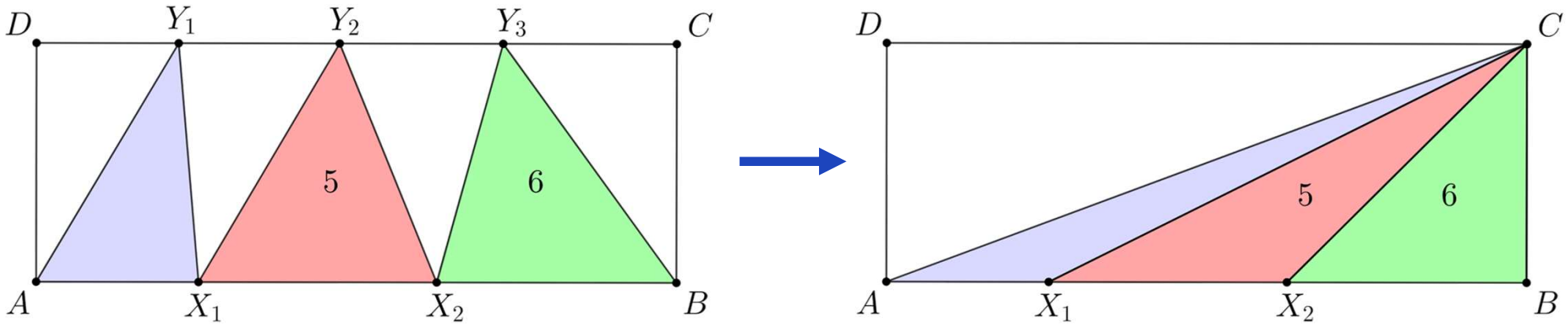
Therefore, $2x + 10 + 12 = 30$.

This gives $x = 4$.

Q1. Triple Spikes

Elegant Solution

Just look at the following picture.



Areas do not change under “sliding”.

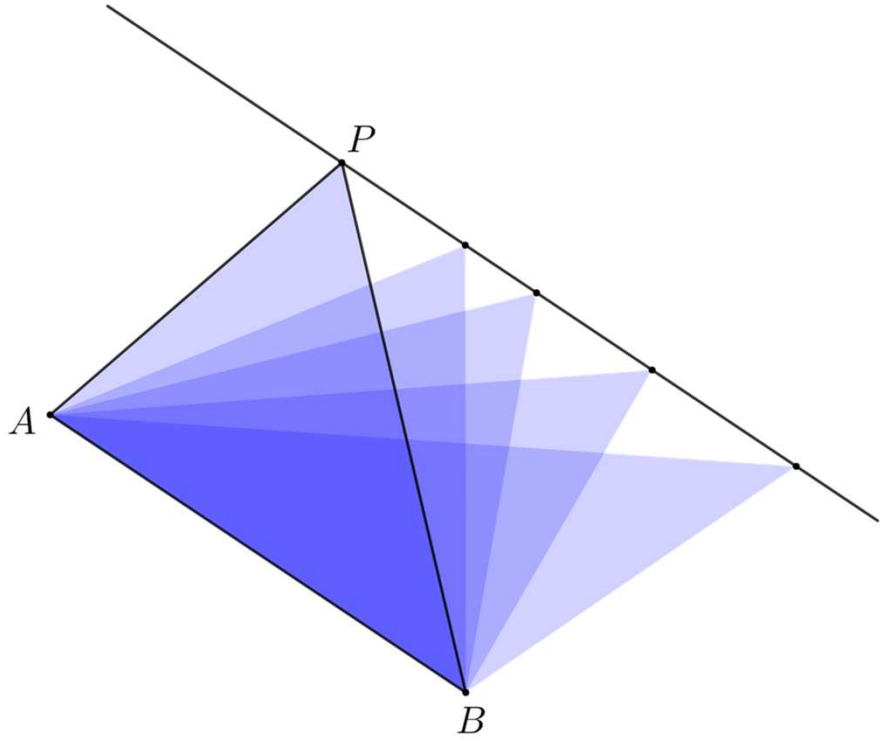
Total area of three triangle is 15. Thus, blue area is $15 - 5 - 6 = 4$.

Sliding Triangle

Let P “move” on the line through P parallel to AB.

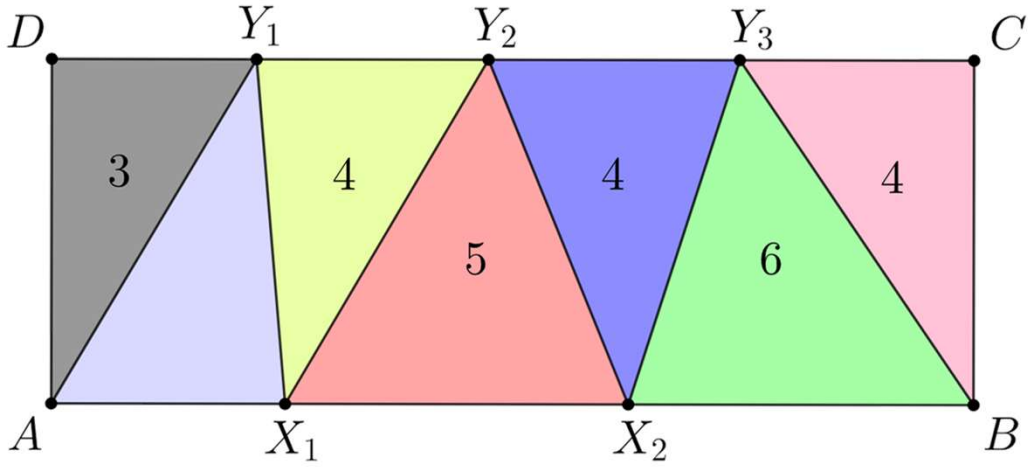
Base and height remain the same.

Therefore, area of APB does not change.



Q2. Sevenfold Spikes

In the picture, ABCD is a rectangle and the areas of the six coloured triangles are as shown in the figure. What is the area of the last coloured triangle?

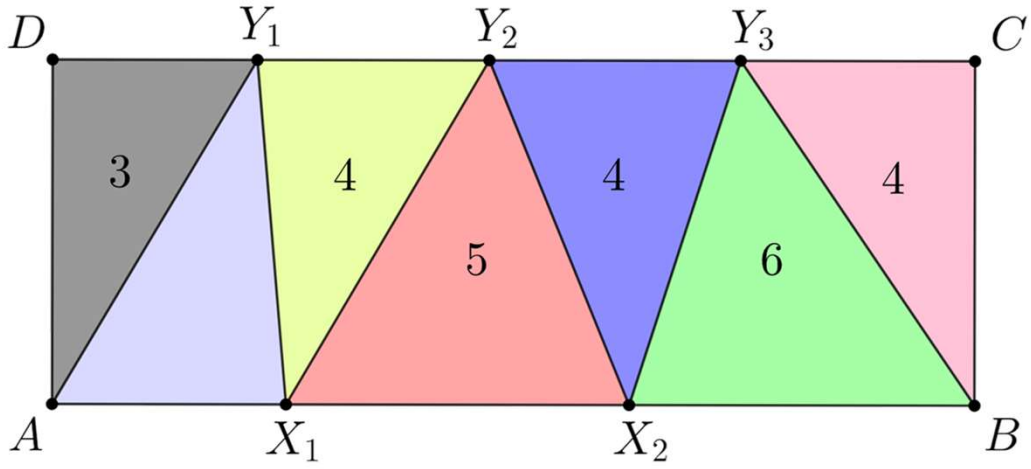


Q2. Sevenfold Spikes

Solution

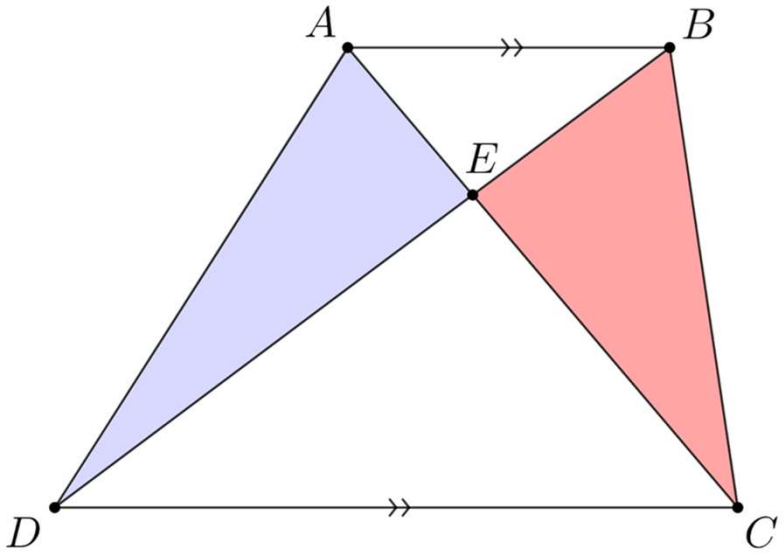
By sliding trick, half of rectangle = area sum of Δ-shape triangles = area sum of ∇-shape triangles.

Therefore, required area is $3 + 4 + 4 + 4 - 5 - 6 = 4$.



Q3. Batwings

In the figure, ABCD is a trapezium with AB || BC. AC and BD meet at E. Prove that areas of EAD and EBC are equal.

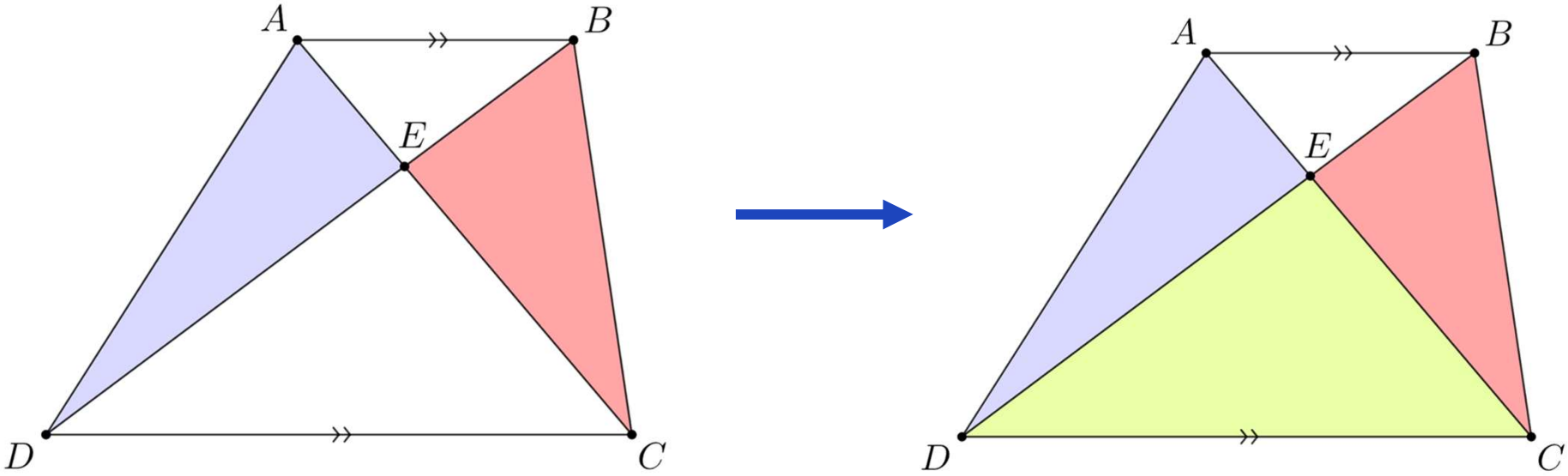


Q3. Batwings

Solution

Areas of triangles ADC and BDC are equal.

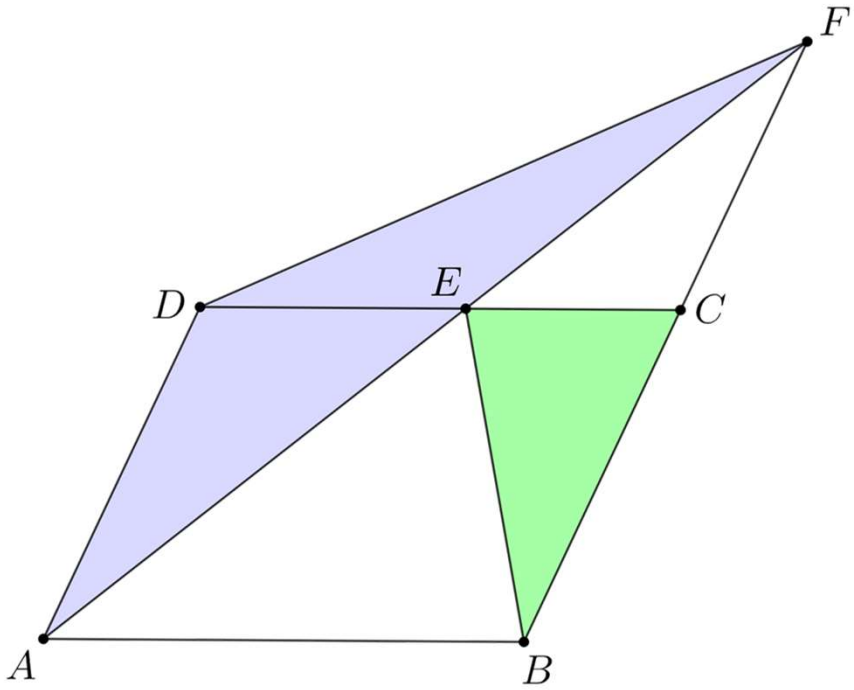
So, blue + yellow = red + yellow. This means that blue = red.



Q4. Parallel Moving

In the figure, ABCD is a parallelogram and A, E, F are collinear. Suppose that DE : EC = 3 : 2.

What is the ratio of blue : green areas?



Q4. Parallel Moving

Solution

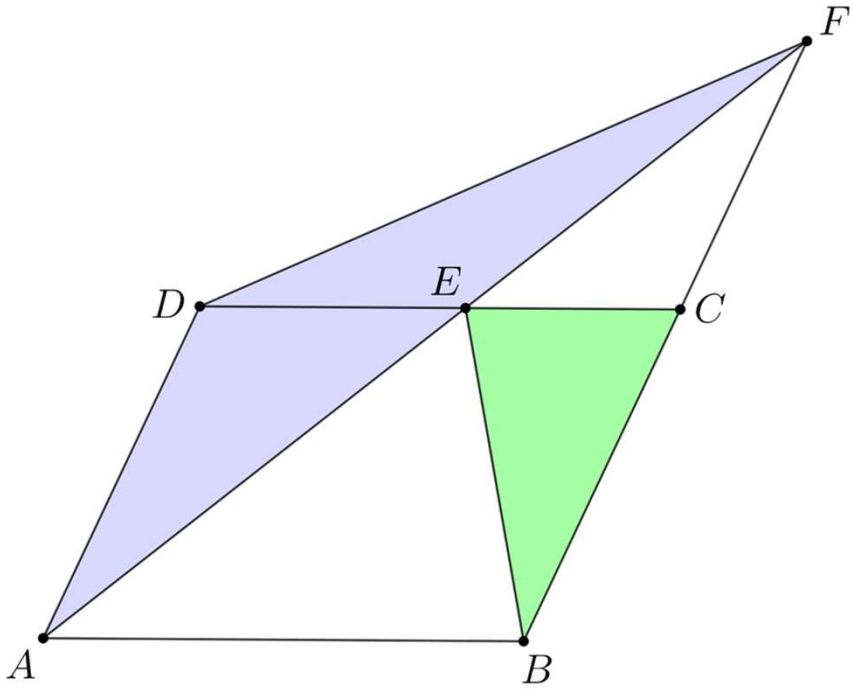
By sliding, blue area = area of ABD = half of ABCD.

Let area of BEC = $2x$.

Then, area of AED = $3x$.

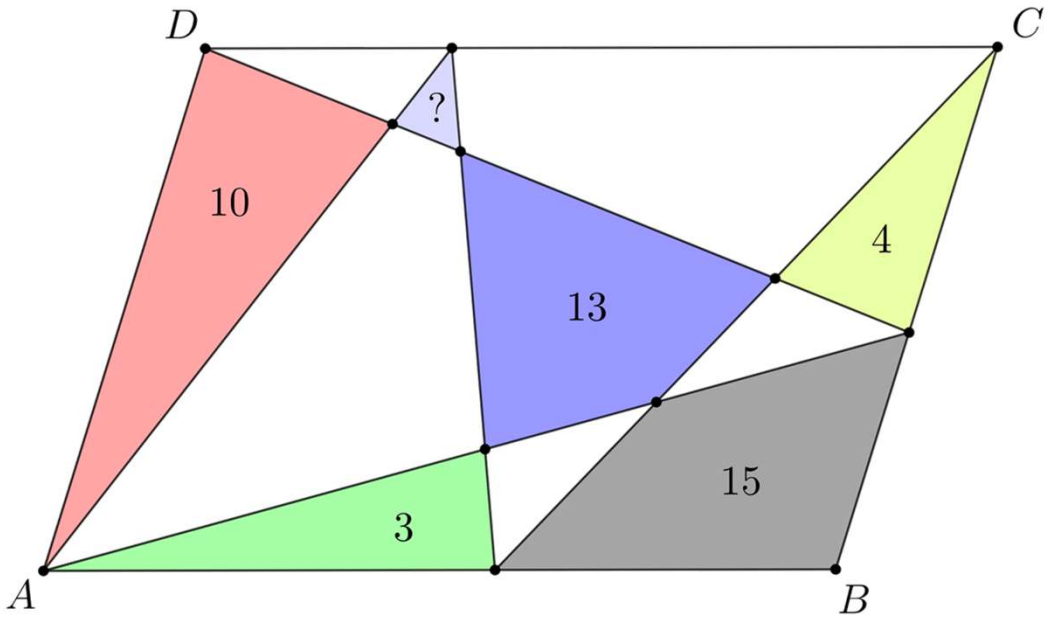
By sliding again, half of ABCD = $2x$ + $3x$.

Therefore, blue : green = 5 : 2.



Q5. Crazy Chinese Problem

In the figure, ABCD is a parallelogram and the areas of little regions are as shown. What is the area of the blue '?' region?

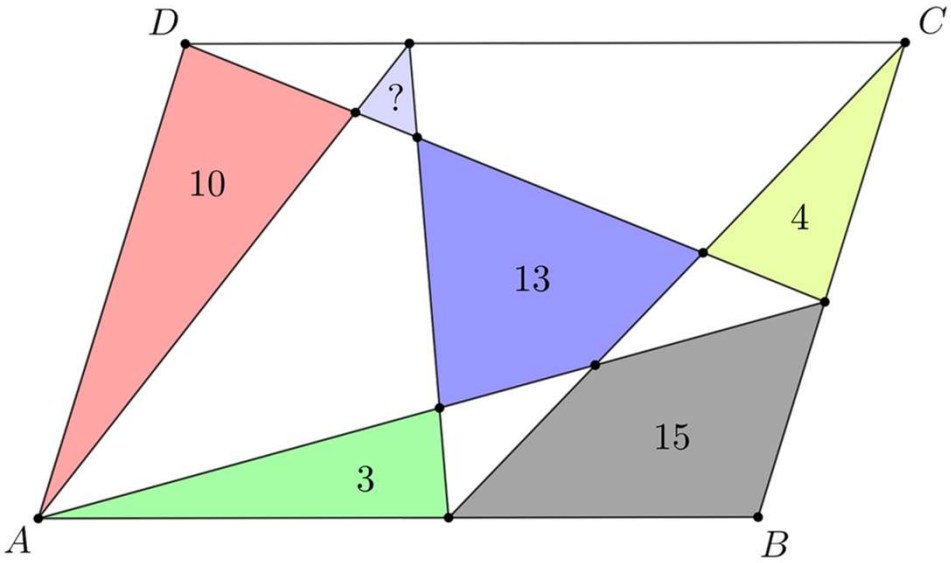


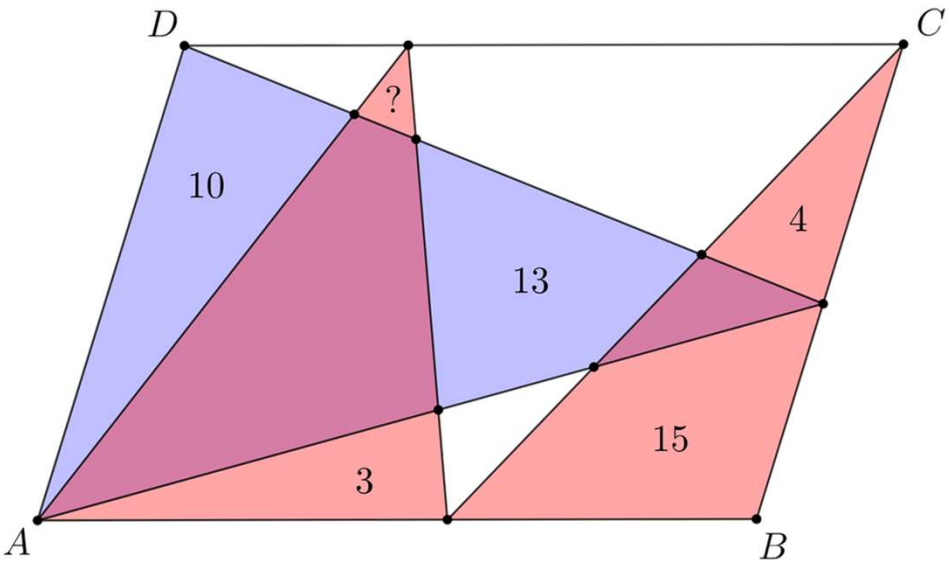
Q5. Crazy Chinese Problem

Solution

In the figure to the right, total blue = total red = half of ABCD.

Thus, ? + 3 + 15 + 4 = 10 + 13.





Section – II

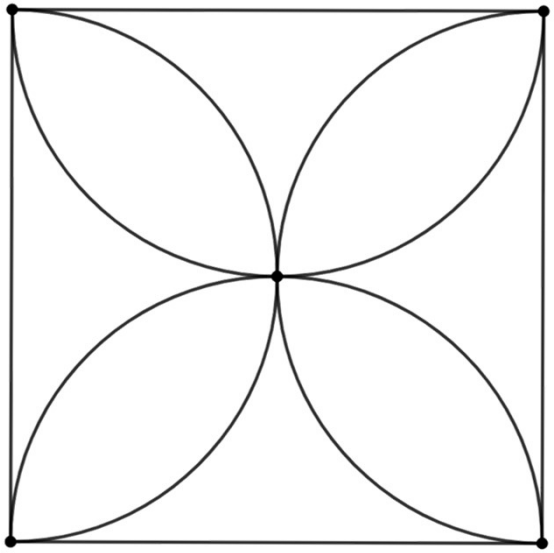
Miscellaneous Calculations

GEO

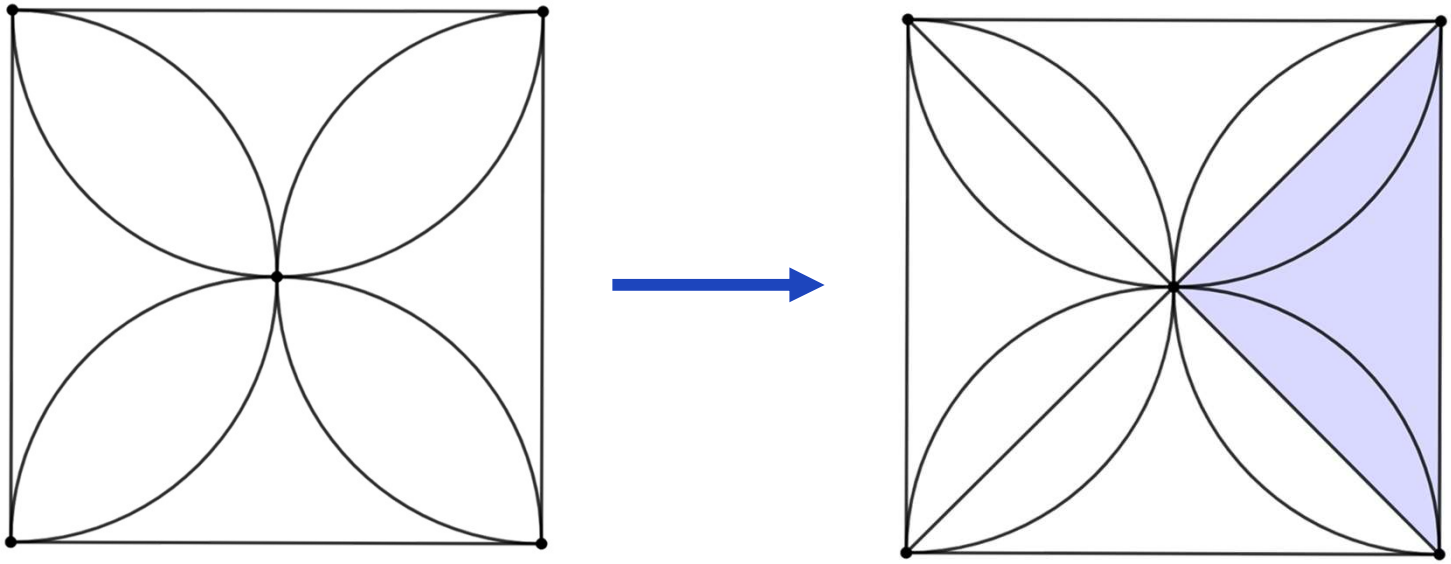
I

Q6. Four Petals

Suppose that the side-length of the square be 4. What is the total area of the four petals?



Q6. Four Petals



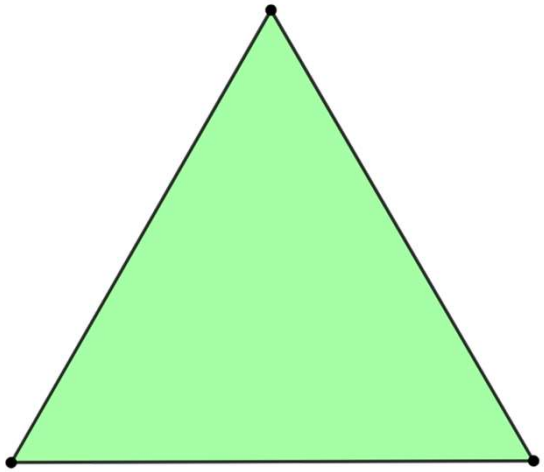
Solution

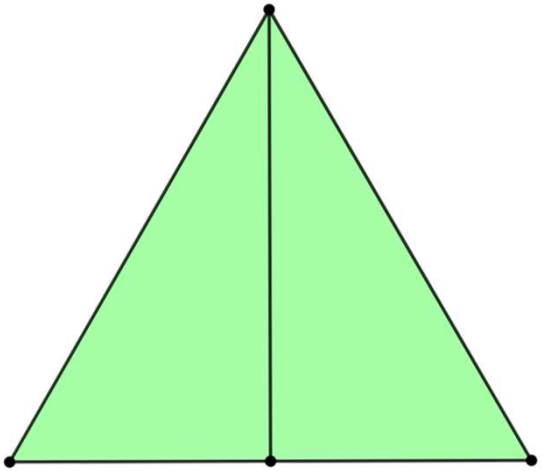
Area of one petal = semicircle − right triangle = $\pi \times 2^2/2 - (4 \times 2)/2 = 2\pi - 4$.

So, total petal area is $8\pi - 16$.

Area of an Equilateral Triangle

Key Idea: Just Pythagoras Theorem





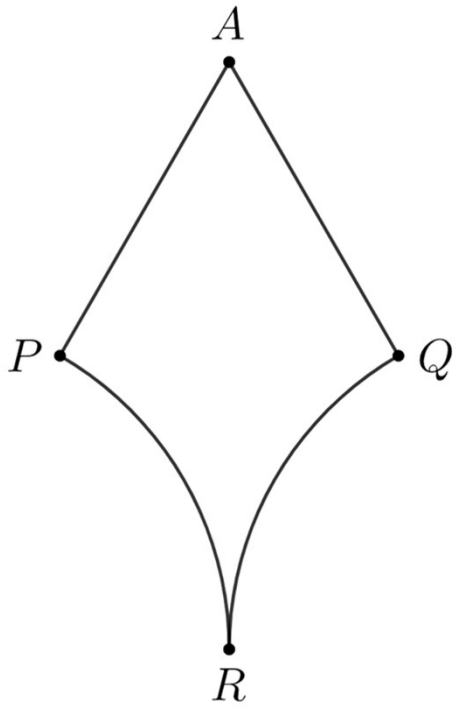
Let s be the side-length. Then, by Pythagoras, height is $\frac{\sqrt{3}}{4}$ s.

Therefore, area is $\frac{\sqrt{3}}{4}$ s^2.

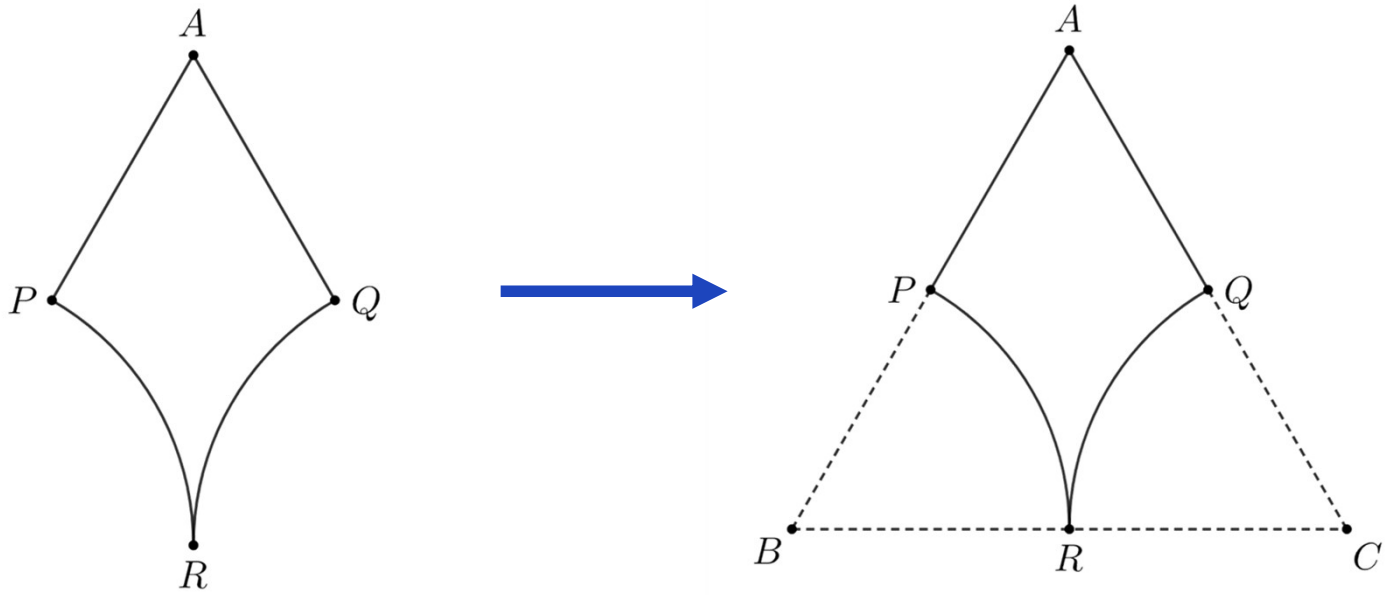
Q7. Stingray

In the figure, angle PAQ is 60 degrees, AP = AQ = 2 and PR, RQ are 1/6 of a circle of radius 2 each.

What is the area of the region show in the figure?



Q7. Stingray



Solution

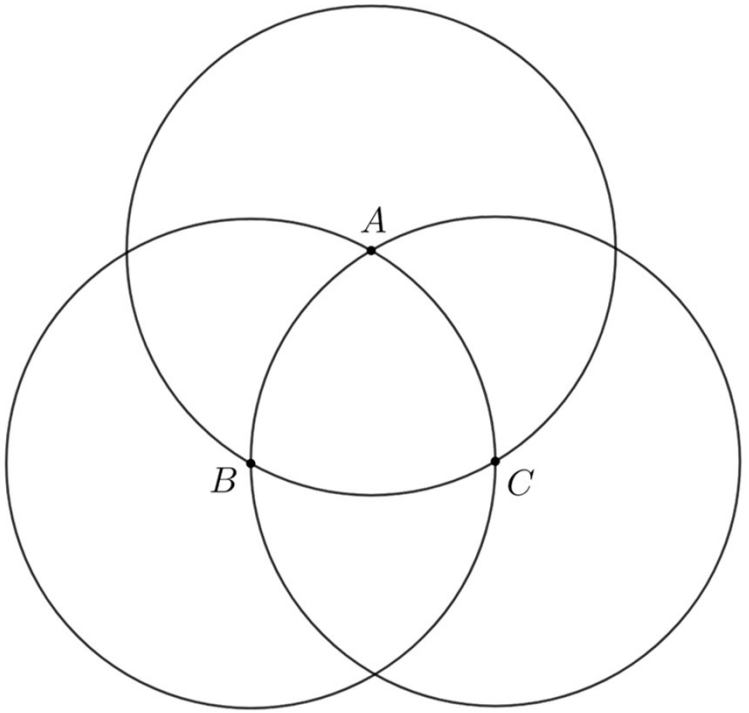
Let B, C be centres of the sectors. Then, ABC is equilateral and P, Q, R are midpoints.

So, required area = area of ABC − 2 × area of sector = $\frac{\sqrt{3}}{4}4^2 - 2 \times \frac{1}{6}\pi 2^2 = 4\sqrt{3} - \frac{4\pi}{3}$.

Q8. Three Circles

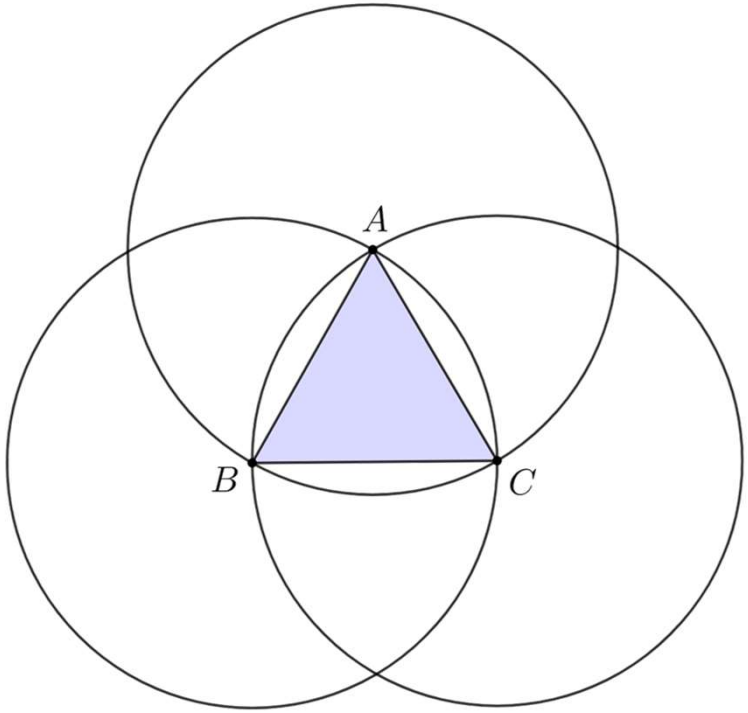
Consider three circles centred at A, B, C each passing through the centres of the others. Suppose these circles have radius 2.

What is the area common to all 3 circles?



Q8. Three Circles

Solution



Triangle ABC is equilateral. The required area is equal to

3 × area of 60° sector − 2 × area of triangle ABC

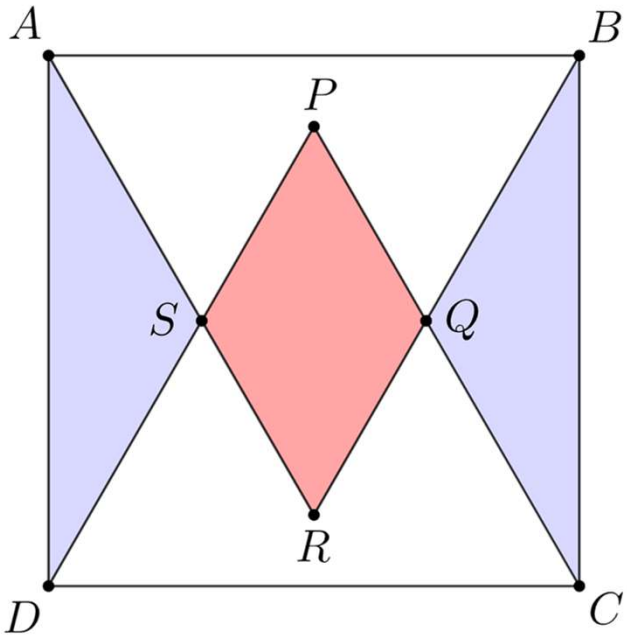
= semicircle area − 2 × area of triangle ABC

$$= \frac{1}{2}\pi 2^2 - 2 \times \frac{\sqrt{3}}{4}2^2$$

$$= 2\pi - 2\sqrt{3}$$

Q9. Red minus Blue

In the figure, ABCD is a square with side length 4, ABR and CDP are equilateral triangles. What is the difference between the red area and the blue area?



Q9. Red minus Blue

Solution

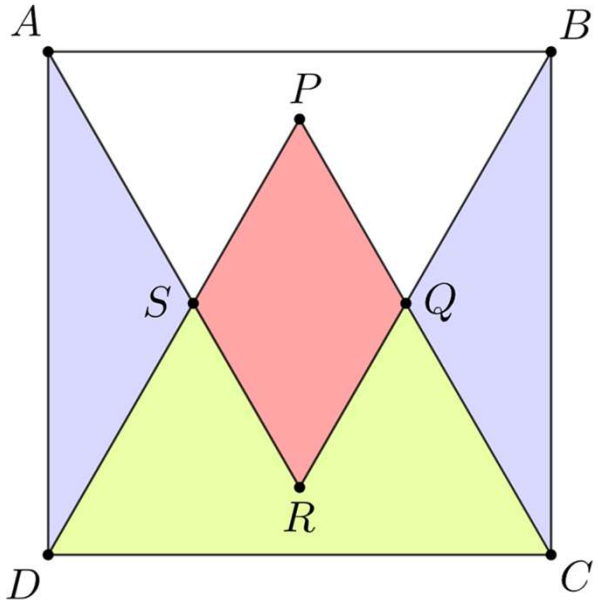
Red – Blue = (Red + Yellow) – (Blue + Yellow).

Red + Yellow is $\frac{\sqrt{3}}{4}4^2 = 4\sqrt{3}$.

Blue + Yellow is $4^2 - \frac{\sqrt{3}}{4}4^2 = 16 - 4\sqrt{3}$.

Therefore, Red – Blue = $8\sqrt{3} - 16$. ← This is negative because $16 = 8 \times 2 = 8\sqrt{4}$

So, difference is $16 - 8\sqrt{3}$.



That's it for this video.

See you in the class!

GEO

I