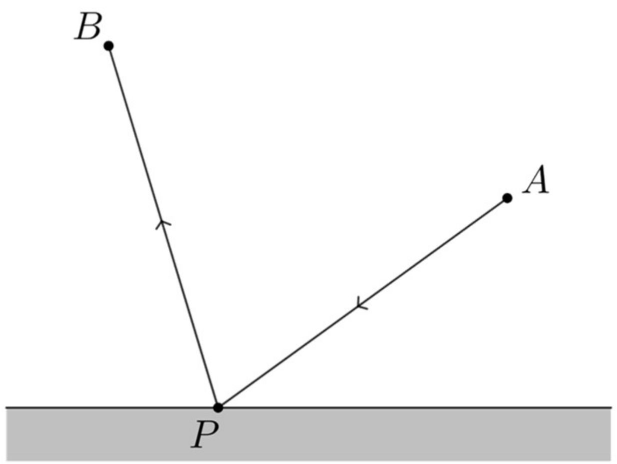
# We will begin at 07:03 PM

Meanwhile, try this problem:

Let A and B be two fixed points above the ground. P is a moving point on the ground. Mr. Light wants to go from A to B by visiting P along the way.

A is 3 ft above the ground, B is 5 ft above the ground and the horizontal distance between A and B is 6 ft.

What is the length of the shortest path that Mr. Light could take?



Record the meeting.

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Lesson – 5

# Congruence and Symmetry

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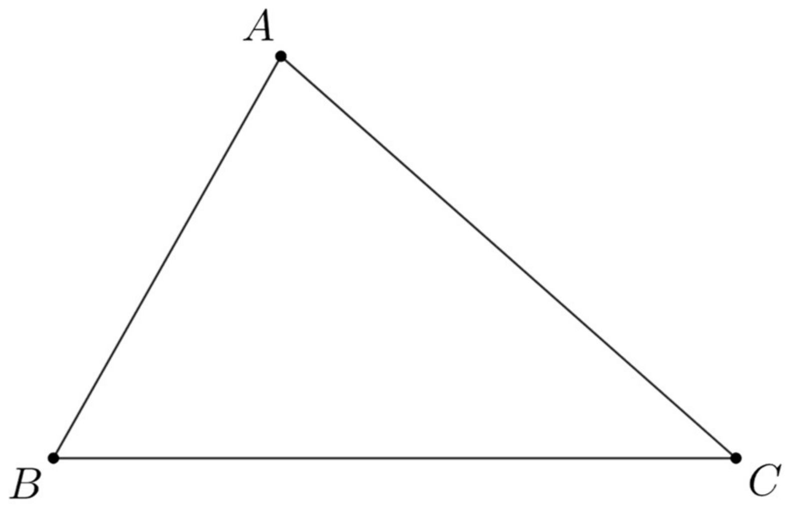
# Help Hla Hla

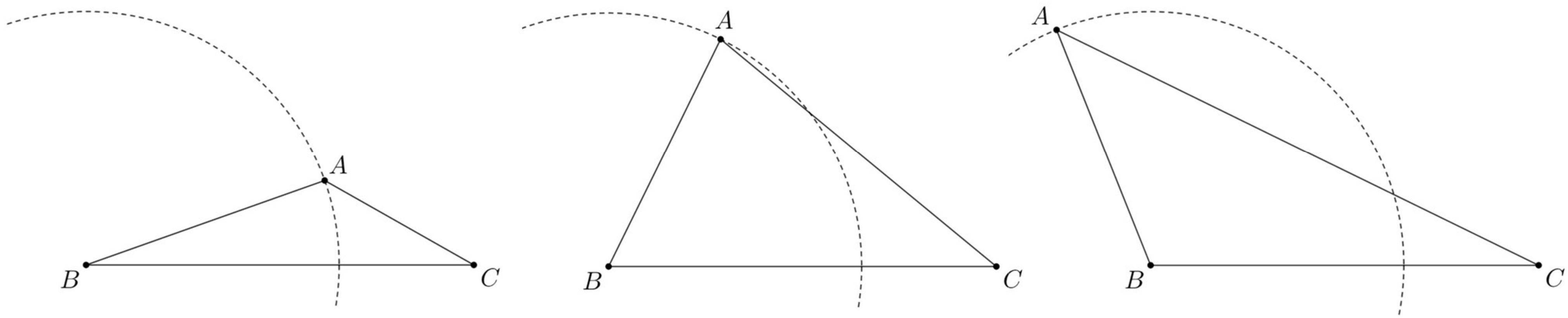
Hla Hla has a triangle ABC. She knows the lengths of BC and AB. Will she be able to determine the remaining side-lengths and angles?



# Help Hla Hla

Answer: No

Reason: Given AB and BC, there are infinitely many possible triangles. Hence, she can't determine a unique solution (she can still determine the range of possible values).



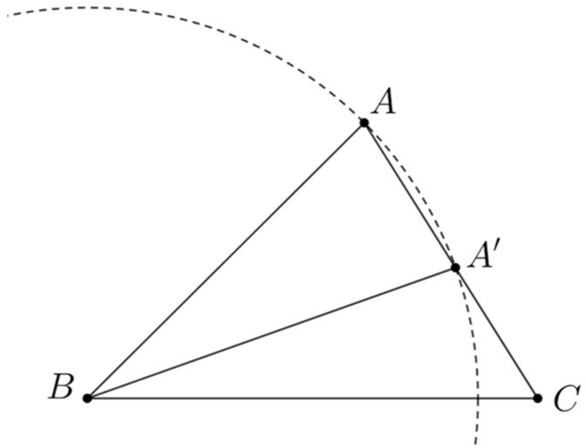

Question: What else does she need to know if she wants a unique answer?

# Help Hla Hla

Correct answers: Angle B (or) Length of AC

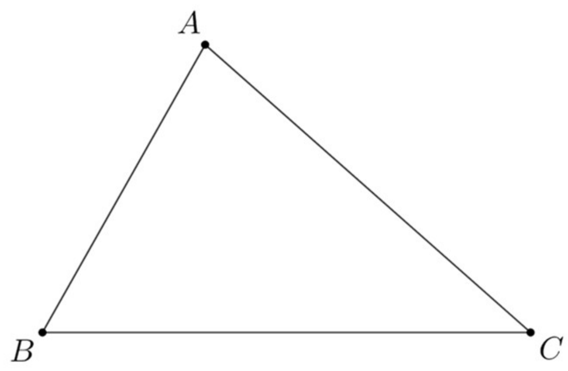
Incorrect answers: Angle A (or) Angle C

Reason: If she knows angle C for example, there are still 2 possibilities for triangle ABC. Same reasoning holds for angle A.

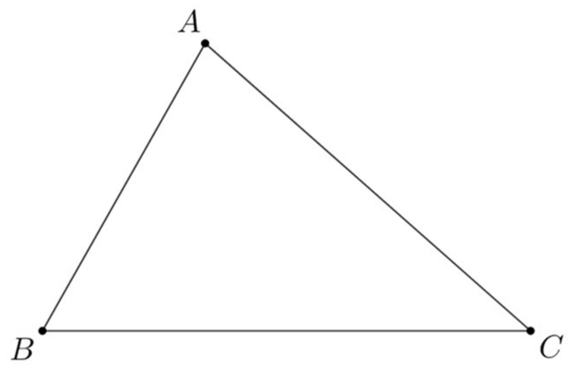


# Triangle Congruence Laws

Let there be two triangles.




SSS: If all 3 pairs of corresponding sides are equal, then two triangles are congruent.



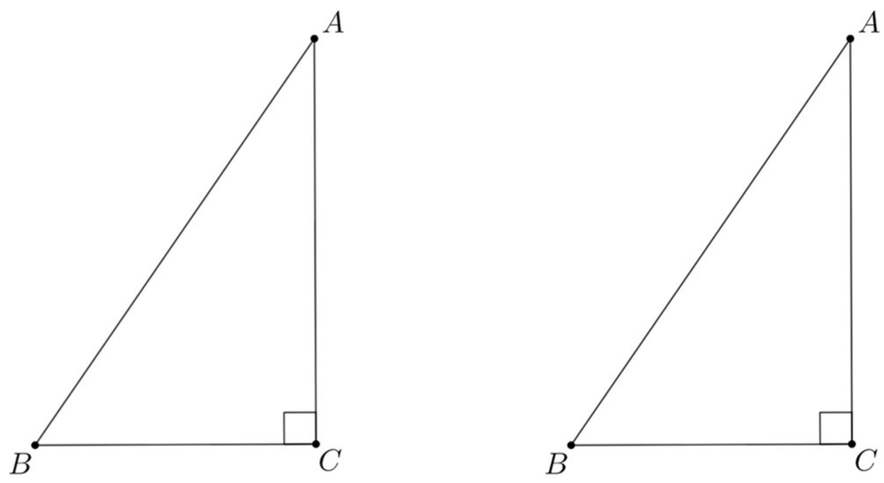

SAS: If 2 pairs of corresponding sides are equal, and the angle between them are also equal, then two triangles are congruent.

Warning: SSA is false i.e. the angle must be between the sides. Remember, SSA can produce two possible triangles.

# Exception for SSA

SSA is correct if the A is a right angle.

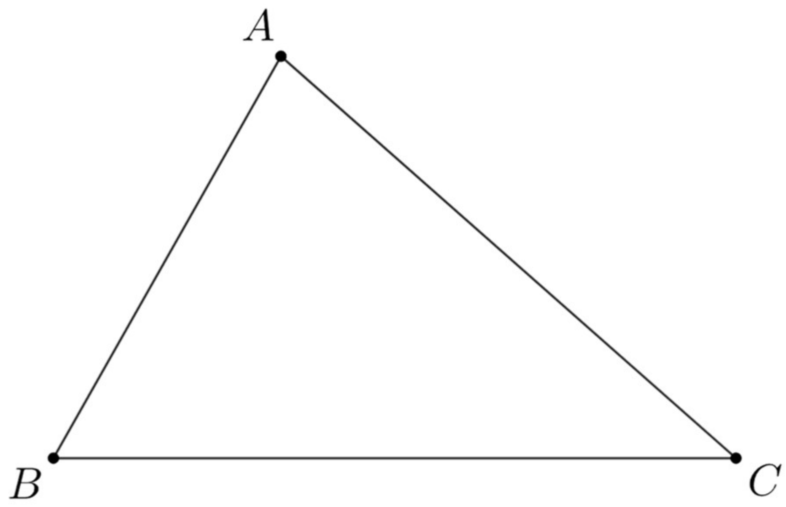
Reason: In this case, the remaining S's must be equal by the Pythagoras Theorem. Therefore, the two triangles must be congruent by SSS.




People usually call this rule as RHS or RHL. It is easier to remember it as SSR.

# Help Hla Hla Again

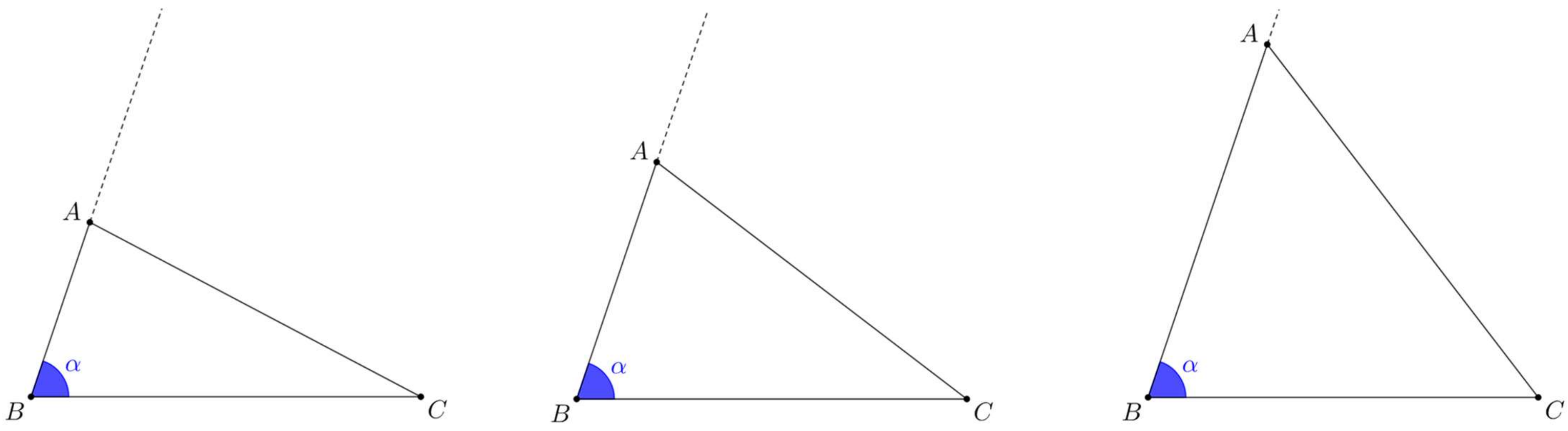
Hla Hla has a triangle ABC. She knows the lengths of BC and angle B. Will she be able to determine the remainging side-lengths and angles?



# Help Hla Hla Again

Answer: No

Reason: Given BC and angle B, there are infinitely many possible triangles. Hence, she can't determine a unique solution (she can still determine the range of possible values).



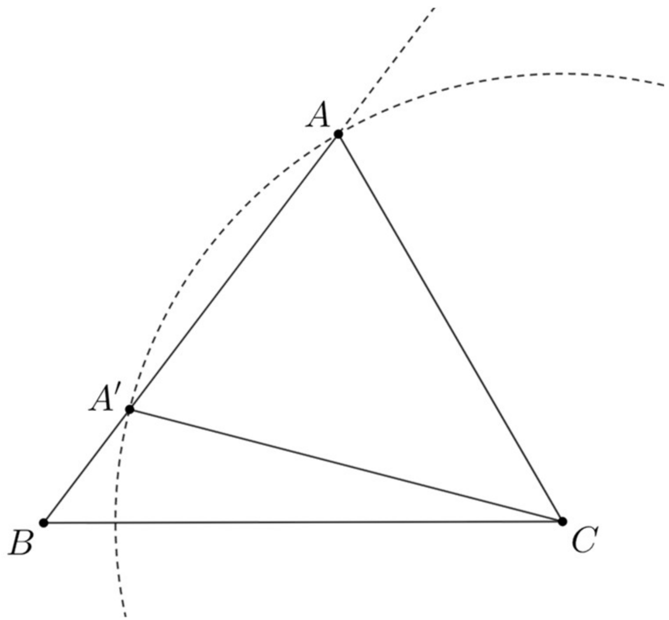

Question: What else does she need to know if she wants a unique answer?

# Help Hla Hla Again

Correct answers: Length of AB (or) Angle C (or) Angle A

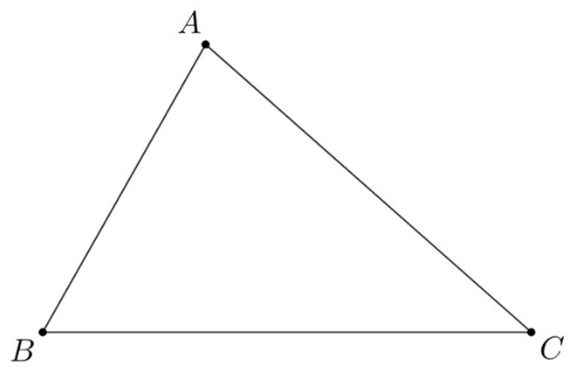
Incorrect answers: Length of AC

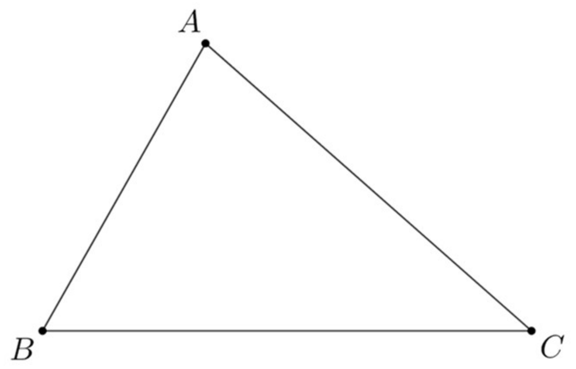
Reason: It can happen that there are two possible locations for A even if she knows AC.



# Triangle Congruence Laws

Let there be two triangles.




SSS: If all 3 pairs of corresponding sides are equal, then two triangles are congruent.

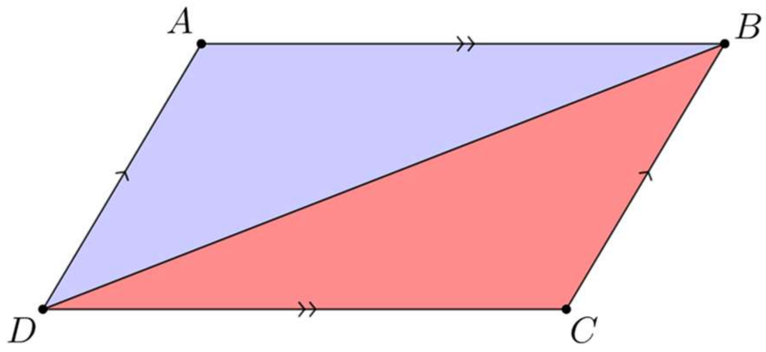
SAS: If 2 pairs of corresponding sides are equal, and the angle between them are also equal, then two triangles are congruent.

SSR (or RHS or RHL): If 2 pairs of corresponding sides are equal, and 1 pair of angles are 90°, then two triangles are congruent.

AAS: If 2 pairs of angles are equal and 1 pair of sides are equal, then two triangles are congruent.

# Q1. Congruence Practice

Why does a diagonal of a parallelogram divide it into two congruent triangles?
Only use the fact that the opposite sides of a parallelogram are parallel.



# Q1. Congruence Practice

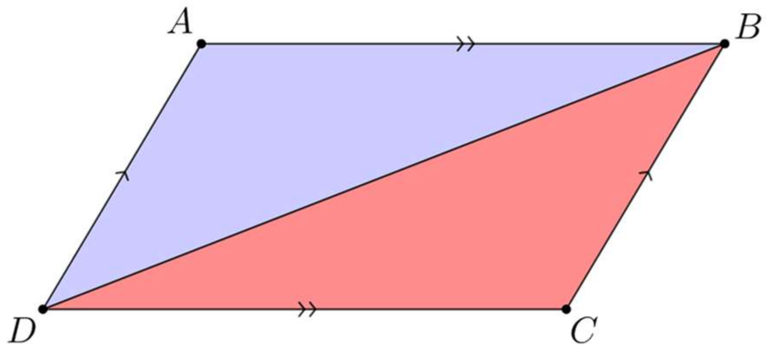
Solution

Because AB || CD, ∠ABD = ∠CDB.

Because AD || BC, ∠ADB = ∠CBD.

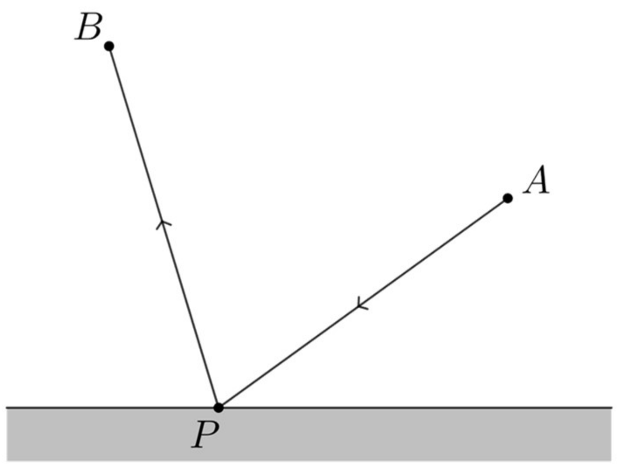
Of course, BD = BD.

Hence, triangles ABD and CDB are congruent by AAS.



# Q2. Shortest Bounce

Let A and B be two fixed points above the ground. P is a moving point on the ground. Mr. Light wants to go from A to B by visiting P along the way.




A is 3 ft above the ground, B is 5 ft above the ground and the horizontal distance between A and B is 6 ft.

What is the length of the shortest path that Mr. Light could take?

# Q2. Shortest Bounce

Solution

Reflect B on the ground to get C.

Then, BP = CP (by triangle congruence).

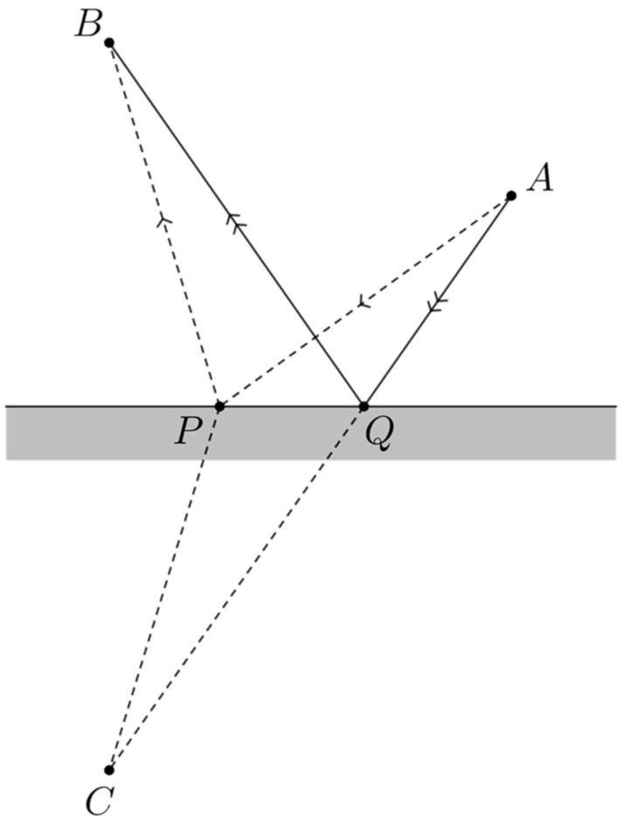
Therefore, minimum value of AP + PB is the same as that of AP + PC.

But, AP + PC is minimum when APC is a straight line.

So, minimum value of AP + PB = length of AC.

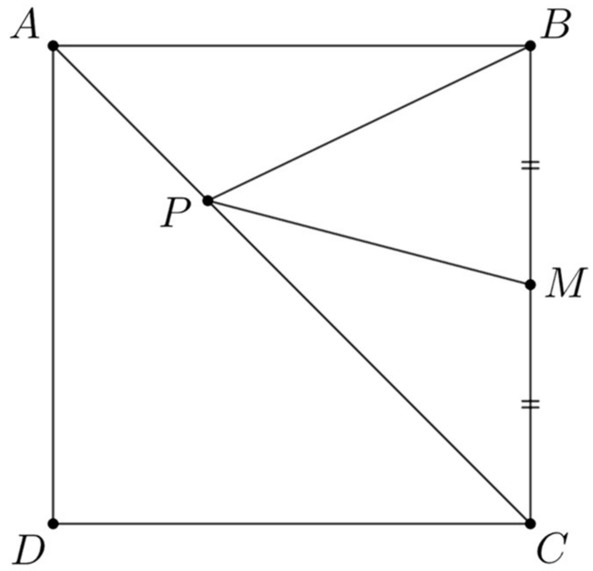
$AC^2 = 6^2 + (3 + 5)^2 = 10^2$, so AC = 10.

Note: In general, PA + PB is minimum when the angle between PA and the ground is equal to that of PB's.



# Q3. Shortest Bounce II

Let ABCD be a square of side-length 10. M is the midpoint of BC. Let P be a variable point on AC. What is the minimum value of MP + PB?



# Q3. Shortest Bounce II

<u>Solution</u>

Triangles APD and APB are congruent (by SAS).

So, PB = PD.

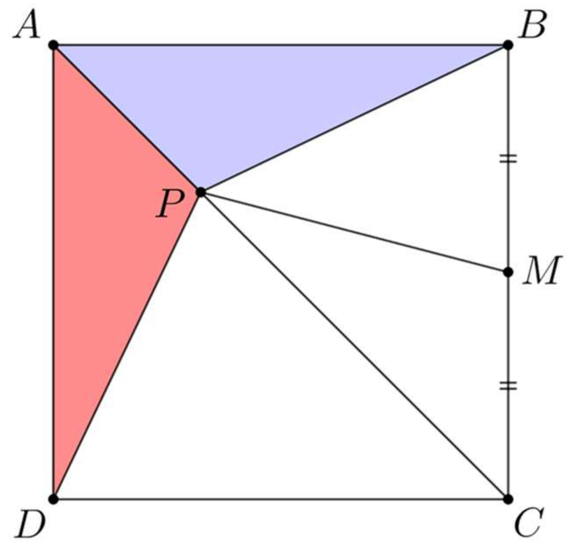
Hence, minimizing MP + PB is the same as minimizing MP + PD.

But, MP + PD is minimized when M, P, D are collinear.

Hence, minimum value of MP + PB = length of MD.

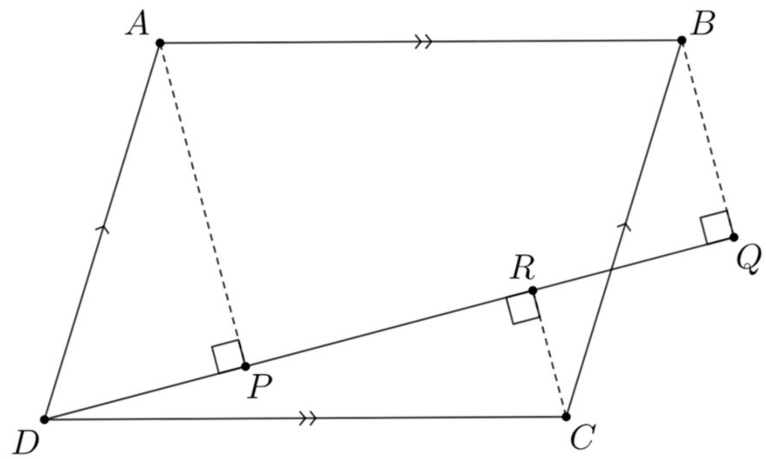
$MD^2 = CM^2 + CD^2 = 10^2 + 5^2 = 125$.

So, MD = 5sqrt(5).



# Q4. Distances from Parallelogram

In the figure, ABCD is a parallelogram. AP, BQ and CR are perpendicular to line DPRQ. Let AP = 9 and CR = 2. What is BQ?



# Q4. Distances from Parallelogram

Solution

Let E be a point such that RQEC is rectangle.

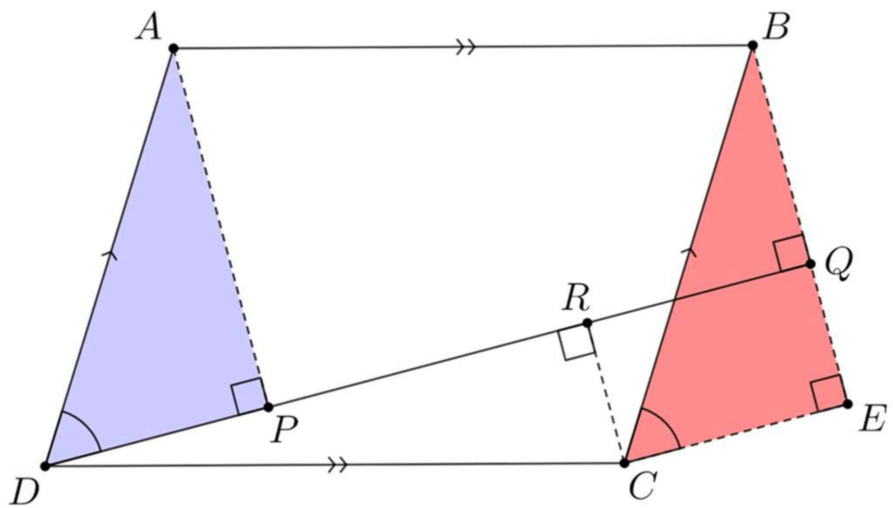
Note that $\angle ADP = \angle BCE$.

Hence, triangles APD and BEC are congruent (by AAS).

Therefore, AP = BE.

So, AP = BQ + CR.

This gives BQ = 9 − 2 = 7.



That's it for this lesson.

Study well for the exam!

GEO

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