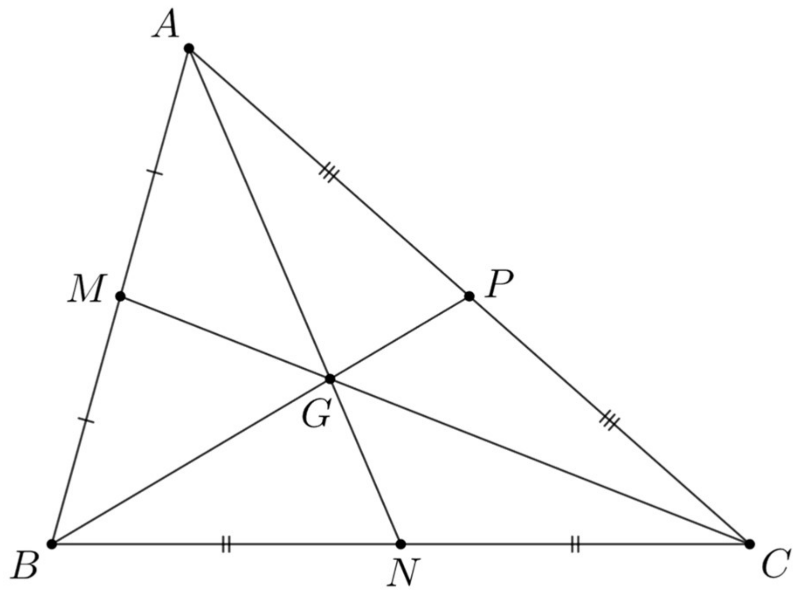
We will begin at 07:03 PM

Try this problem in the mean time:

Three medians AN, BP and CM of triangle ABC meet at point G. Prove that AG = 2GN.



Record the meeting.

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Lesson – 4

Ratios and Similarity

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Equal Ratios Theorem

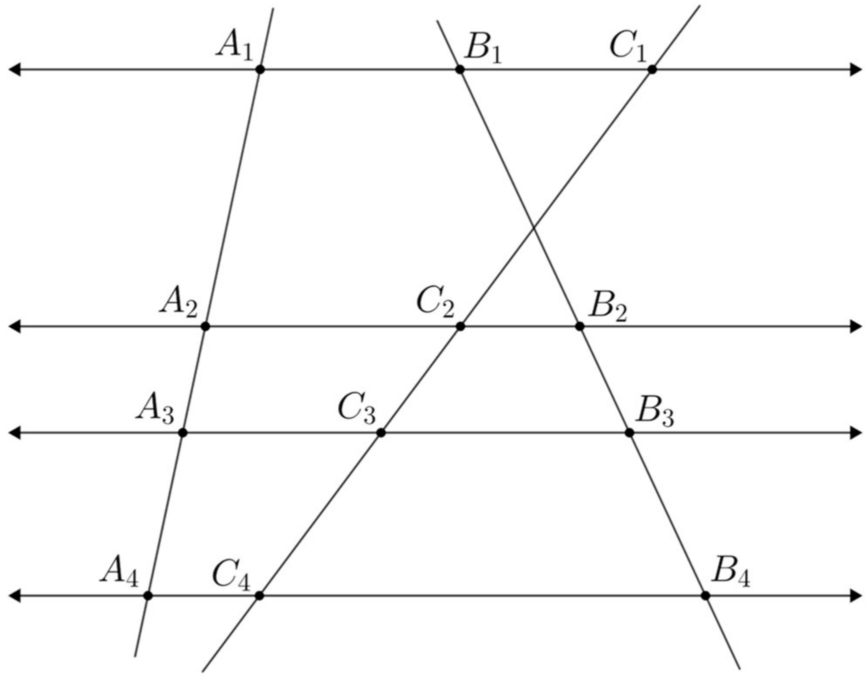
Theorem: Parallel lines always cut the given lines into pieces of equal proportion.

For example, in the figure,

$$A_1A_2 : A_2A_3 = B_1B_2 : B_2B_3 = C_1C_2 : C_2C_3$$

$$A_2A_4 : A_1A_4 = B_2B_4 : B_1B_4 = C_2C_4 : C_1C_4$$

etc.



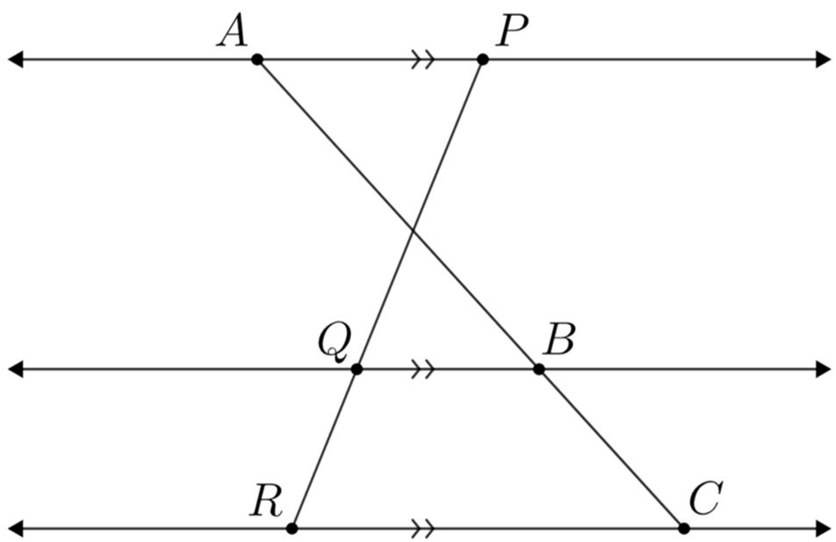
Q1. Ratio Practice I

In the figure, AP || BQ || CR. Suppose that PQ/QR = 3/2. Find

(a) AB : BC

(b) PR : PQ

(c) AC : BC



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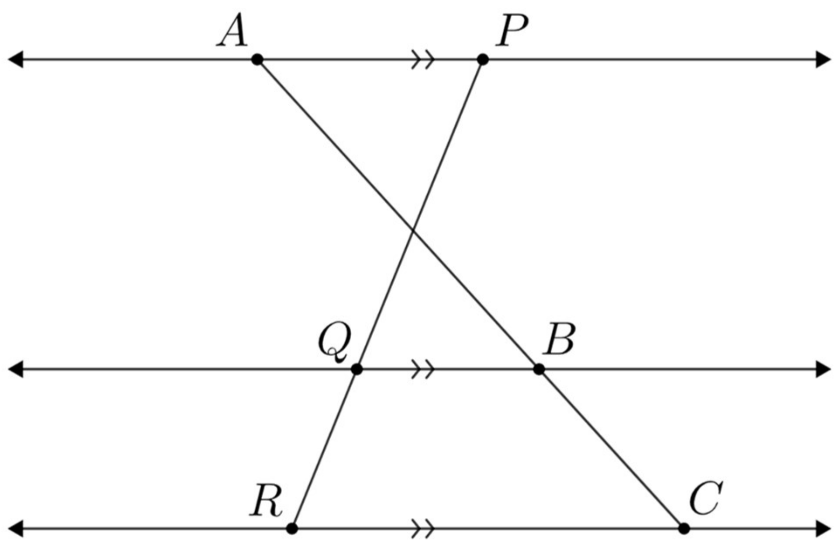


Solution

Let PQ = 3k and QR = 2k.

(a) AB : BC = PQ : QR = 3 : 2.

(b) PR : PQ = 5k : 3k = 5 : 3.

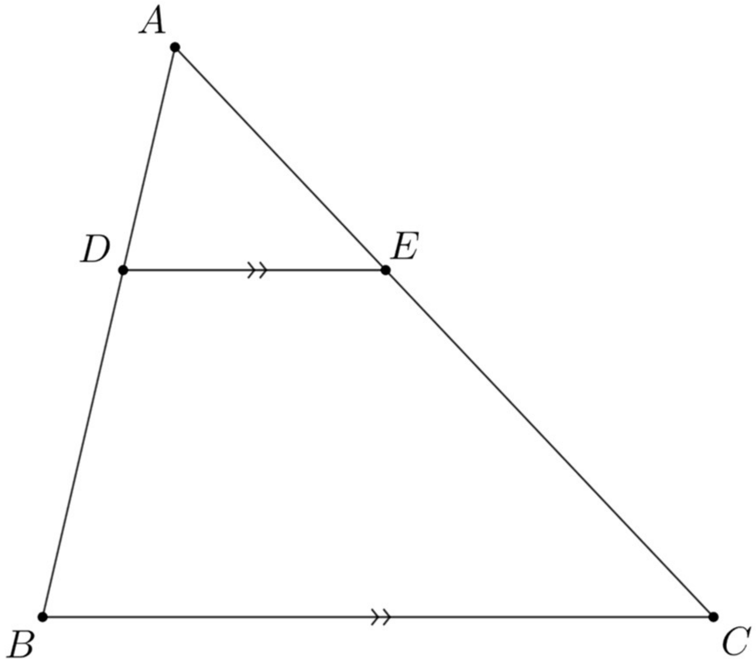
(c) AC : BC = PQ : QR = 5k : 2k = 5 : 2.

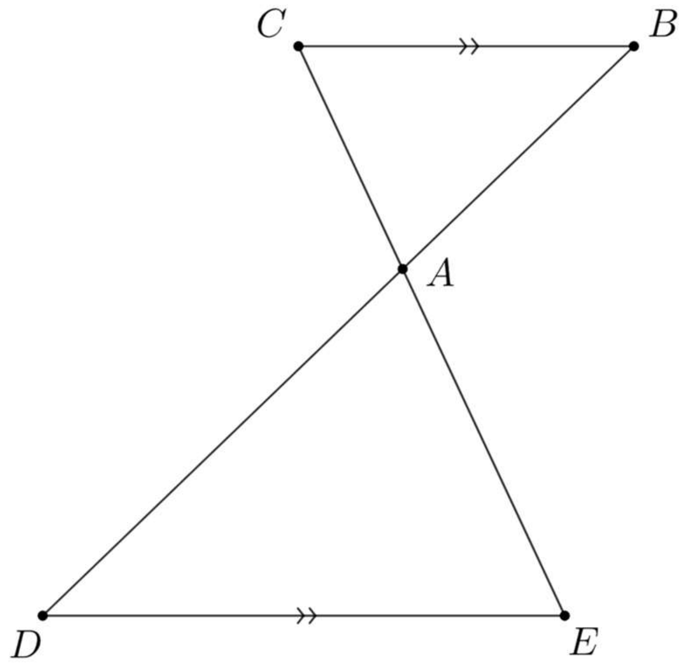
Important Consequence

Theorem: Let DE and BC be parallel lines. Suppose BD and CE cut at A. Then,

$$AD : AB = AE : AC = DE : BC.$$

The converse is also true.





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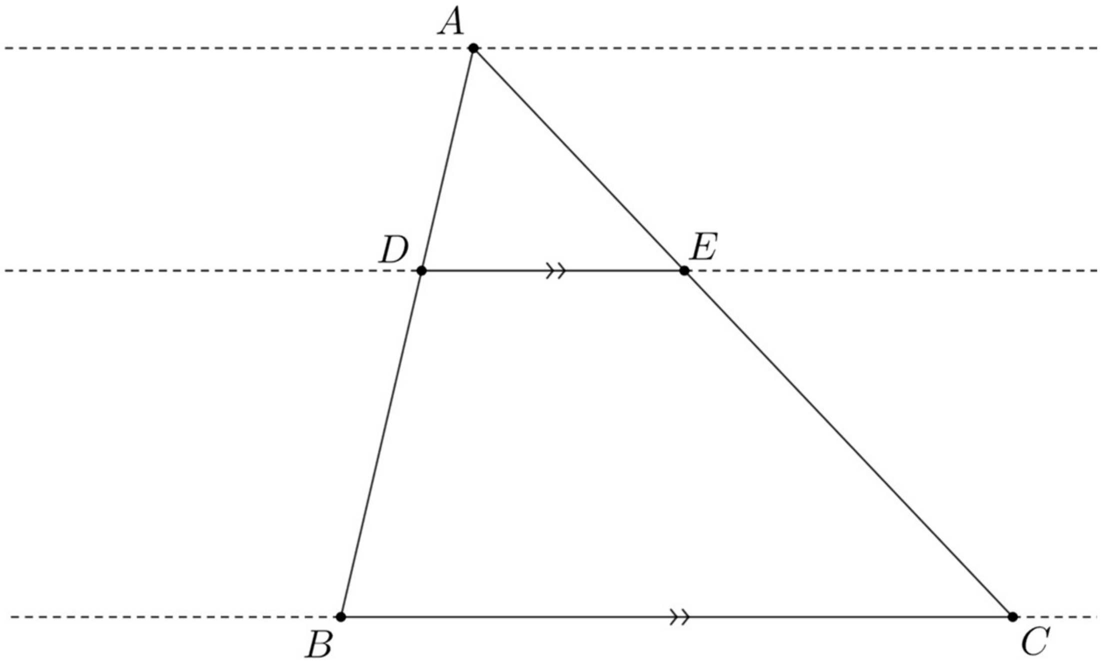
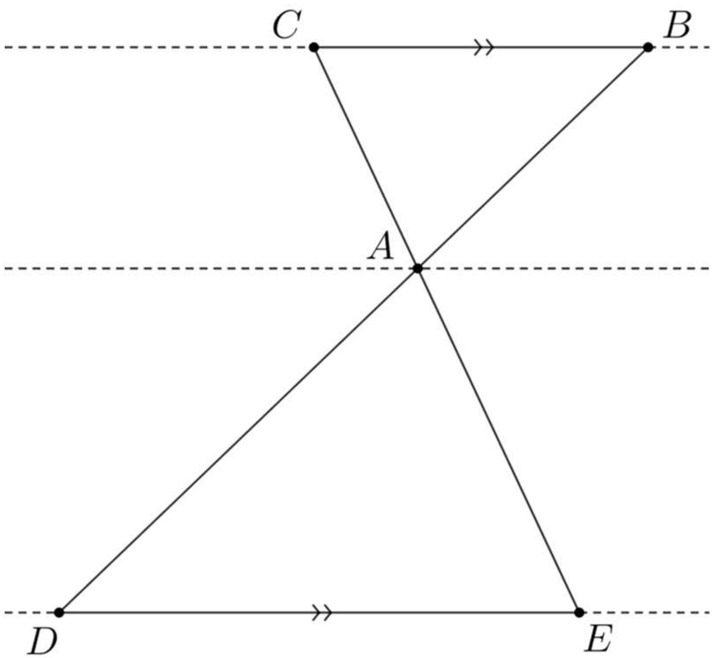




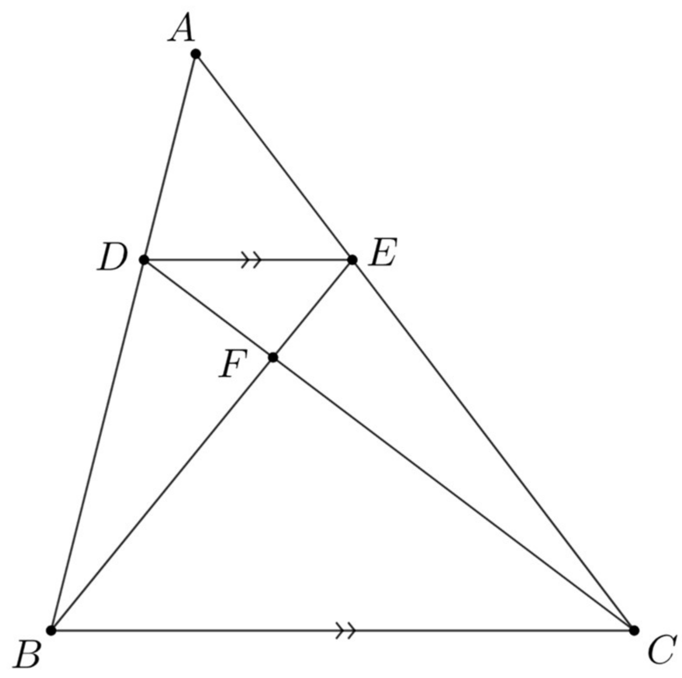
Q2. Ratio Practice II

In the figure, DE || BC. Let DE : BC = 1 : 3.

Find the following ratios:

(a) BF : FE

(b) AD : DB



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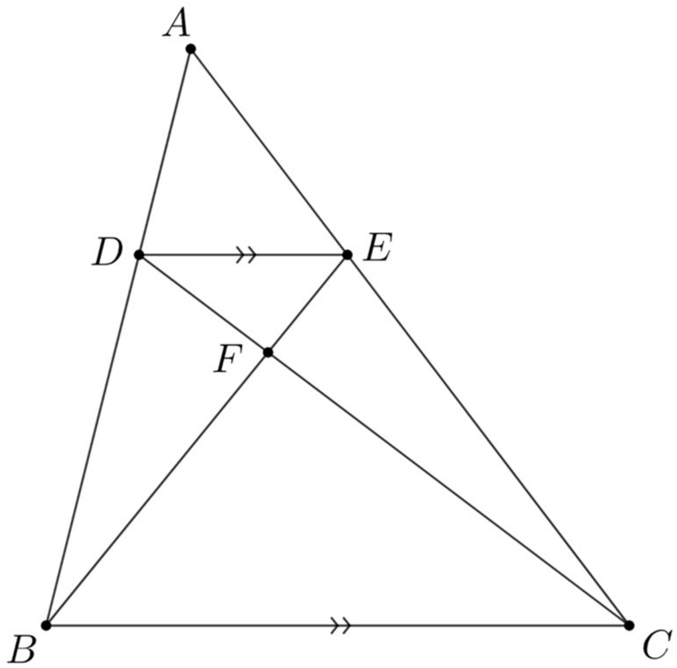


Solution

(a) BF : FE = BC : DE = 3 : 1.

(b) AD : AB = DE : BC = 1 : 3.

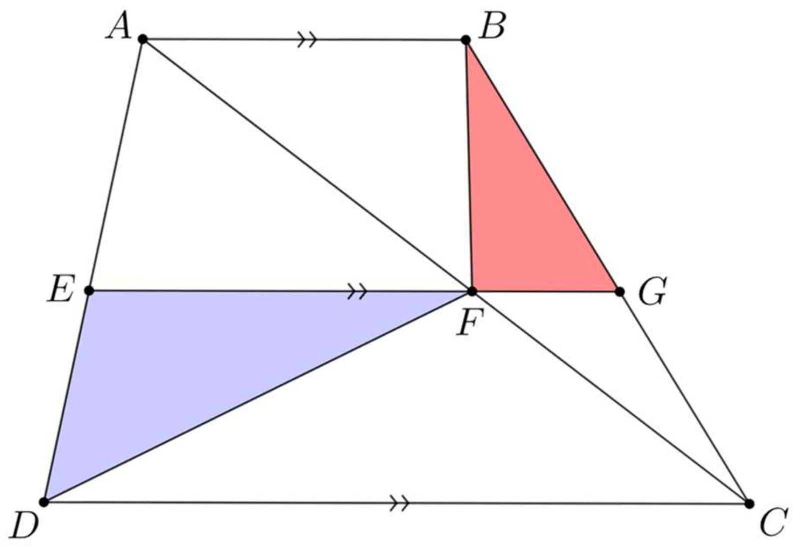
So, let AD = k and AB = 3k. Then, BD = 2k.

Thus, AD : DB = k : 2k = 1 : 2.

Note: If you know a ratio between 3 collinear points, you can find all the ratios within these 3 points. ← Super useful

Q3. Blue and Red

In the figure, AB || EG || CD. Area of triangle DEF (blue) is 10 and area of triangle BGF (red) is 6. Suppose that AF : CF = 2 : 1. Find the area of the trapezium.



Q3. Blue and Red

Solution

From BG : GC = 2 : 1, we get [CGF] = 3.

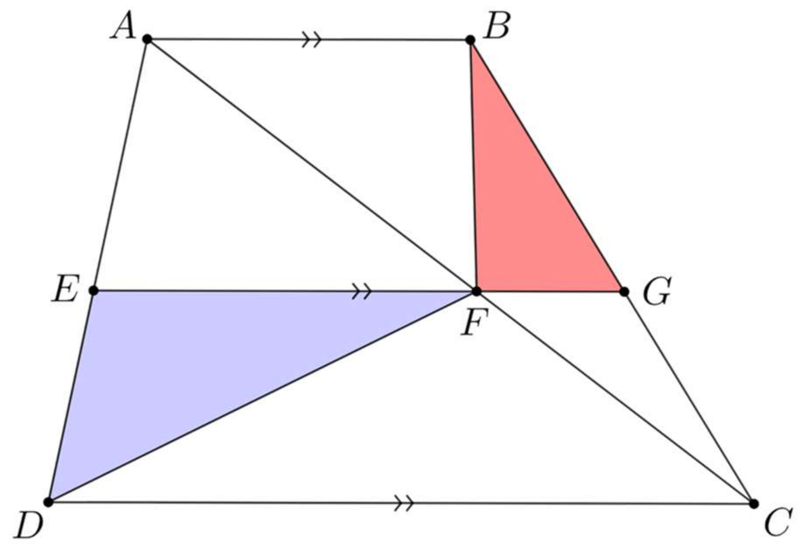
From AF : FC = 2 : 1, we get [ABF] = 18.

From AE : ED = 2 : 1, we get [AEF] = 20.

From AF : FC = 2 : 1, we get [FCD] = 15.

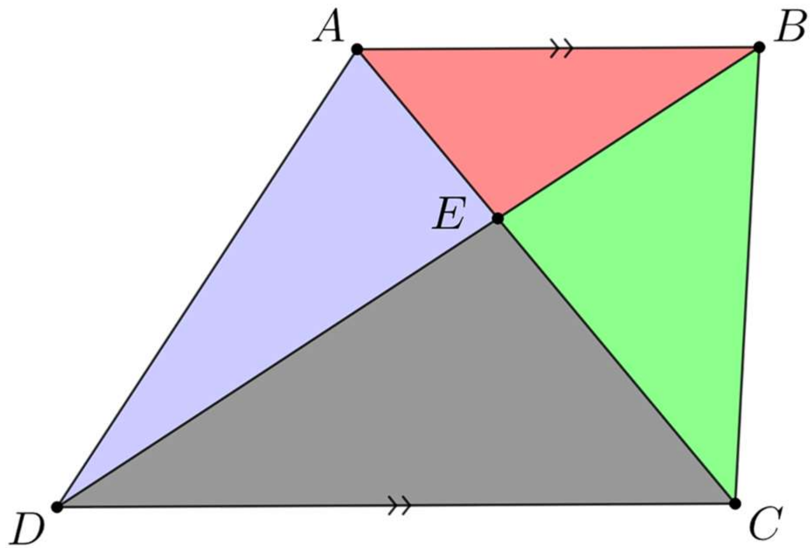
Therefore,

area of ABCD = 3 + 18 + 20 + 15 + 10 + 6 = 72.



Q4. Trapezium Circus

ABCD is a trapezium with AB || CD and area 16. Suppose that AB : CD = 2 : 3. What is the area of triangle DEC (grey)?



Q4. Trapezium Circus

Solution

DE : DB = AE : EC = AB : CD = 2 : 3.

From DE : EB = 2 : 3, we get blue : red = 3 : 2.

So, let blue = 3k and red = 2k.

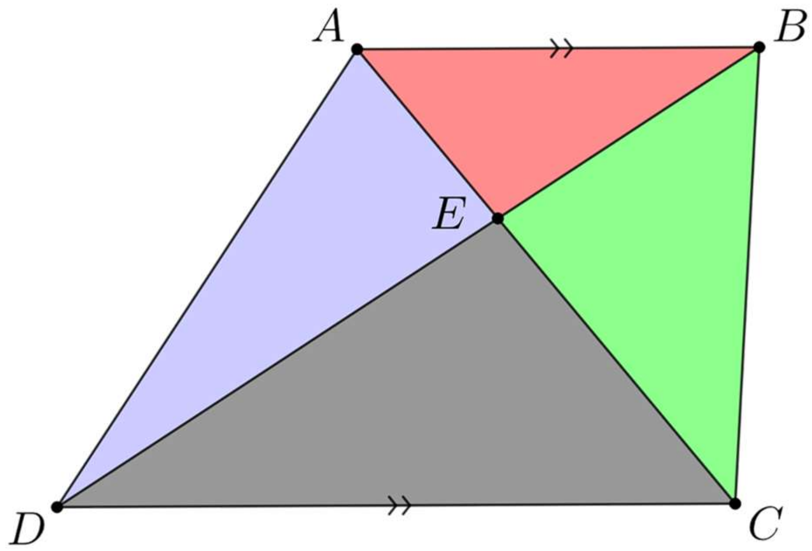
From AE : EC = 2 : 3, we get blue : grey = 2 : 3.

So, grey = 3k $\times$ 3/2 = 9k/2.

From AE : EC = 2 : 3, we get green = 3k.

So, 3k + 3k + 2k + 9k/2 = 16 and hence k = 16/25.

Therefore, grey = 144/25.



Let's have a short break.

We will continue after 5 minutes.

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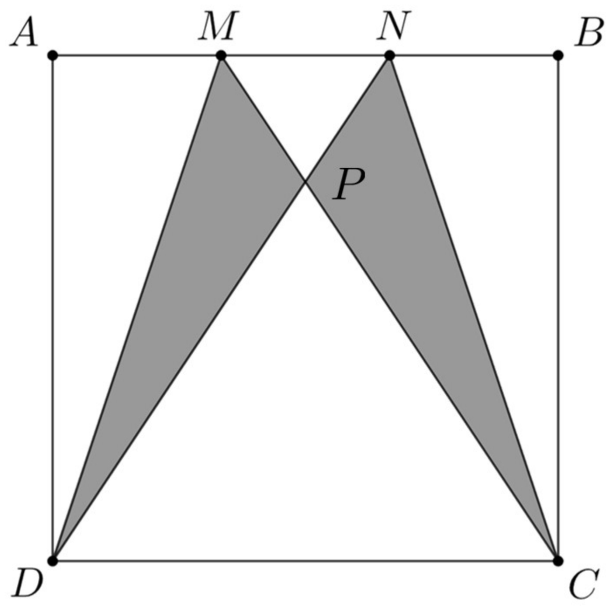
Q5. Batwings

In the figure, ABCD is a square with side length 3 and

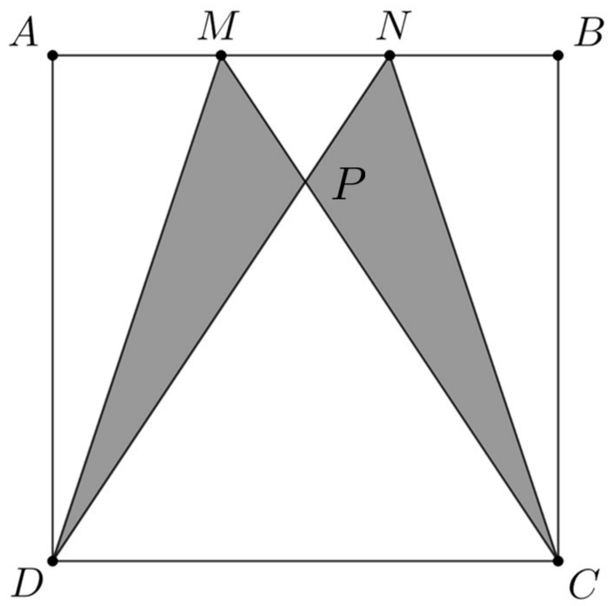
AM = MN = NB.

What is the area of the batwings (grey region)?



Q5. Batwings

Solution



Let area of PDM = 3k.

Then,

Area of PDM : Area of PNM = DP : PN = 3 : 1.

Therefore, area of PNM = k.

Hence, Area of DMN = 4k.

So, $4k = 1 \times 3 / 2$ and hence $k = 3/8$.

Thus,

Area of batwings = $2 \times 3k = 6 \times 3/8 = 9/4$.

I guess we both earned our rest.

See you soon!

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