We will start at 07:05 PM

Try this problem in the mean time:

A ship starts at point O. It then sails 1 km east, 2 km north, 3 km west, 4 km south, 5 km east, ... , 10 km north. What is the distance beween the ship and O?

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Lesson – 3

Right Triangles and Pythagoras

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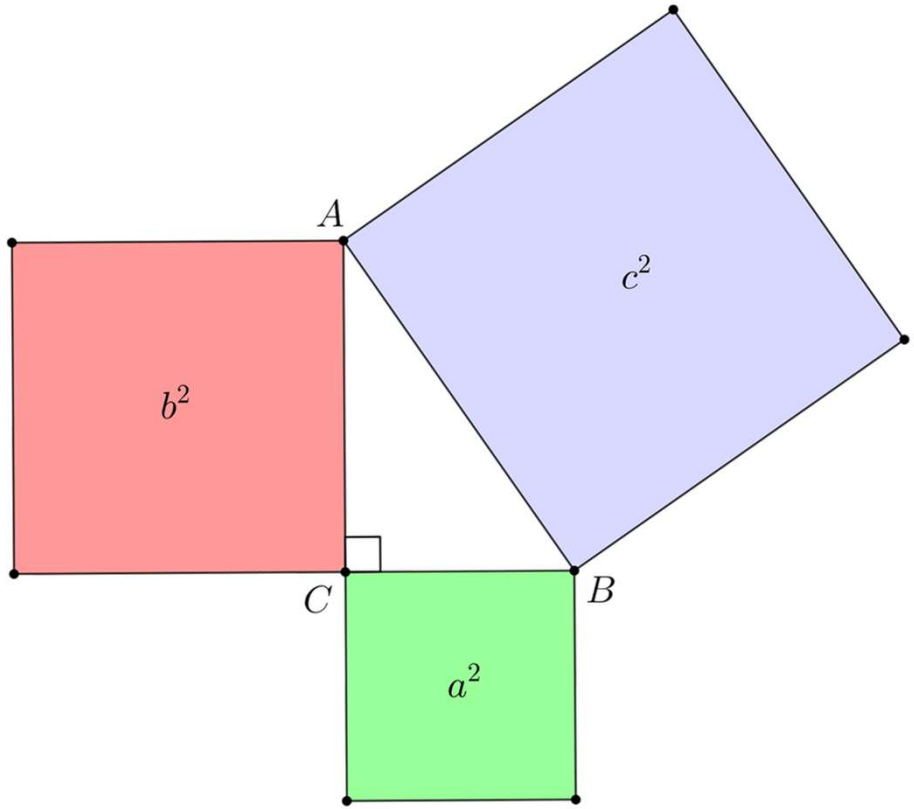
Pythagoras Theorem

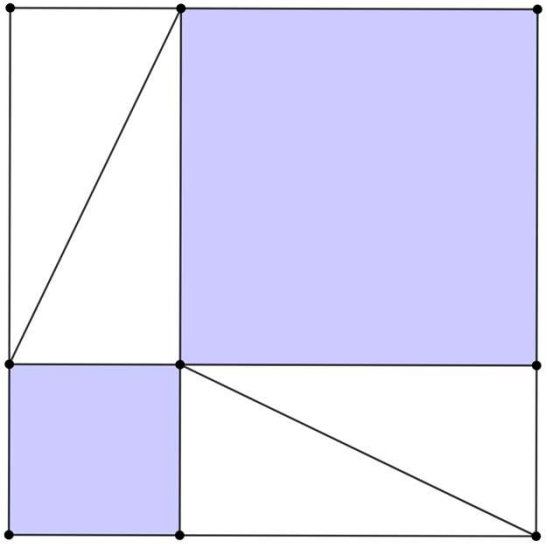
Theorem: Let ΔABC be a right triangle with $\angle C = 90°$. Then, $AB^2 = AC^2 + BC^2$.

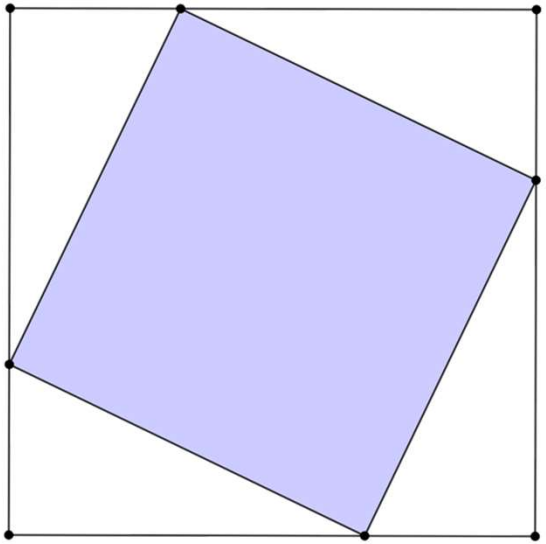


Pythagoras Theorem

Proof

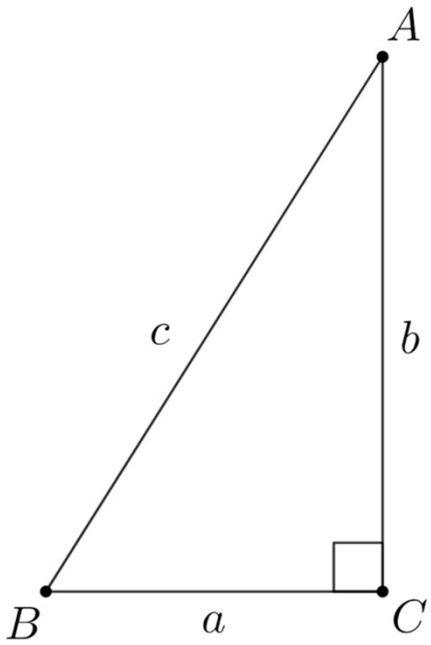
Make 4 copies of ΔABC and assemble them in two different ways as follows. In the left picture, blue area = $AC^2 + BC^2$. In the right picture, blue area = AB^2.





Pythagorean Triples

It is useful to remember some common right triangles with integer side-lengths. For example,



a	b	c
3	4	5
5	12	13
8	15	17
9	40	41

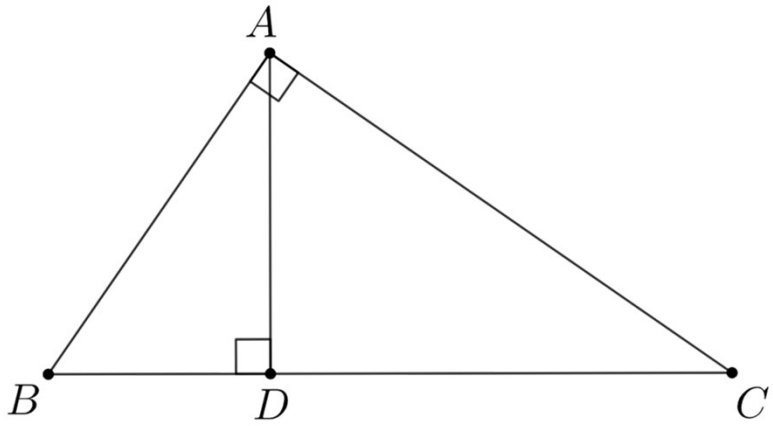
Note that if (a, b, c) form a Pythagorean triple, then (ka, kb, kc) is also a Pythagorean triple for any positive integer k.

For example, (9, 12, 15) and (16, 30, 34) are Pythagorean triples.

Q1. Warm-up

Let ABC be a right triangle with $\angle A = 90°$.

Let BD = 6, AB = 10 and CD = 15. Find AC.



Solution

$AD^2 = 10^2 - 6^2 = 64$

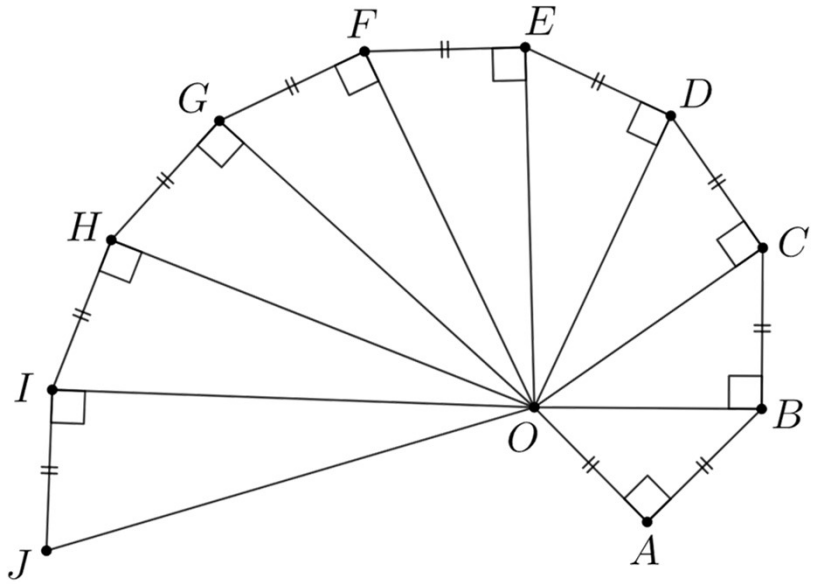
$AC^2 = 64 + 15^2 = 289$

Therefore, AC = 17.

Q2. Pythagorean Spiral

In the figure, all the marked lengths are equal to 1.

What is the length of OJ?



Q2. Pythagorean Spiral

In the figure, all the marked lengths are equal to 1.

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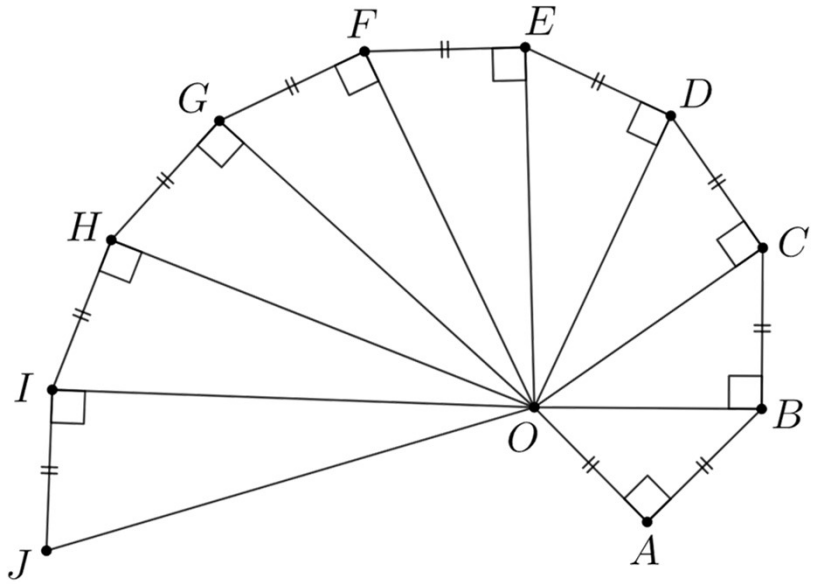


Solution

$OB^2 = 1^2 + 1^2 = 2$

$OC^2 = 2 + 1^2 = 3$

$OD^2 = 3 + 1^2 = 4$

$OE^2 = 4 + 1^2 = 5$

...

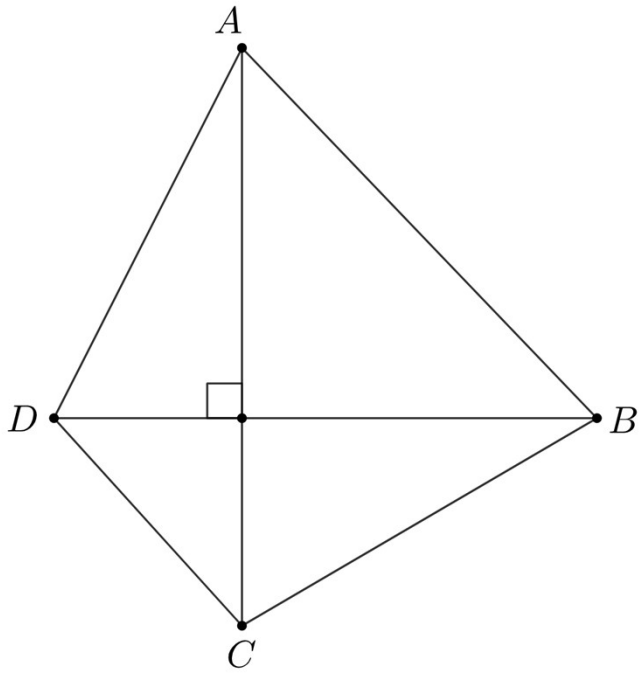
$OJ^2 = 9 + 1^2 = 10$

Hence, OJ = sqrt(10).

Q3. Perpendicular Diagonals

Diagonals of a convex quadrilateral ABCD are perpendicular.

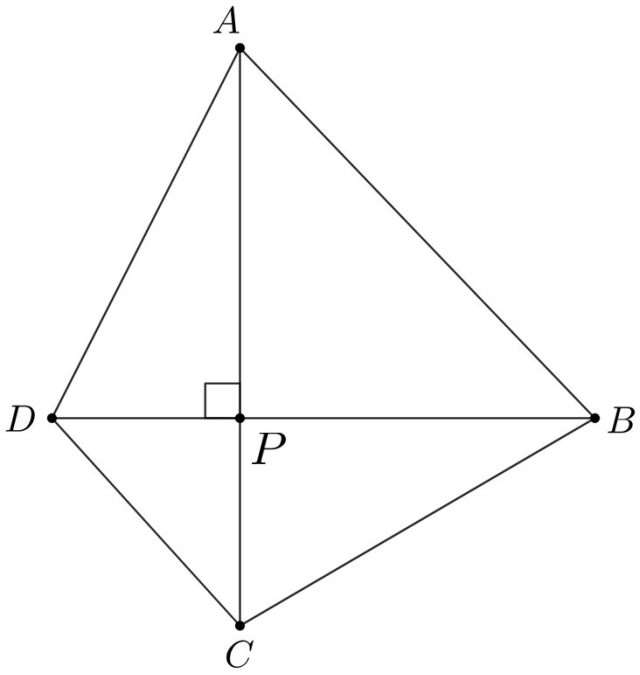
Suppose that AB = 7, BC = 6 and CD = 4. Find AD.



Q3. Perpendicular Diagonals

Diagonals of a convex quadrilateral ABCD are perpendicular.

Suppose that AB = 7, BC = 6 and CD = 3. Find AD.



Solution

Let two diagonals cut at P.

$AD^2 = PA^2 + PD^2$

$PA^2 = AB^2 - PB^2$

$PD^2 = CD^2 - PC^2$

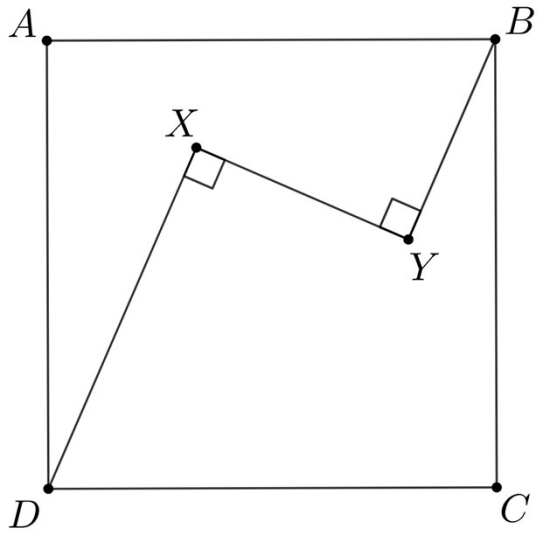
So,

$AD^2 = AB^2 + CD^2 - PB^2 - PC^2 = AB^2 + CD^2 - BC^2$.

Hence, $AD^2 = 49 + 9 - 36 = 22$. So, AD = sqrt(22)

Q4. Zig-zag in Square

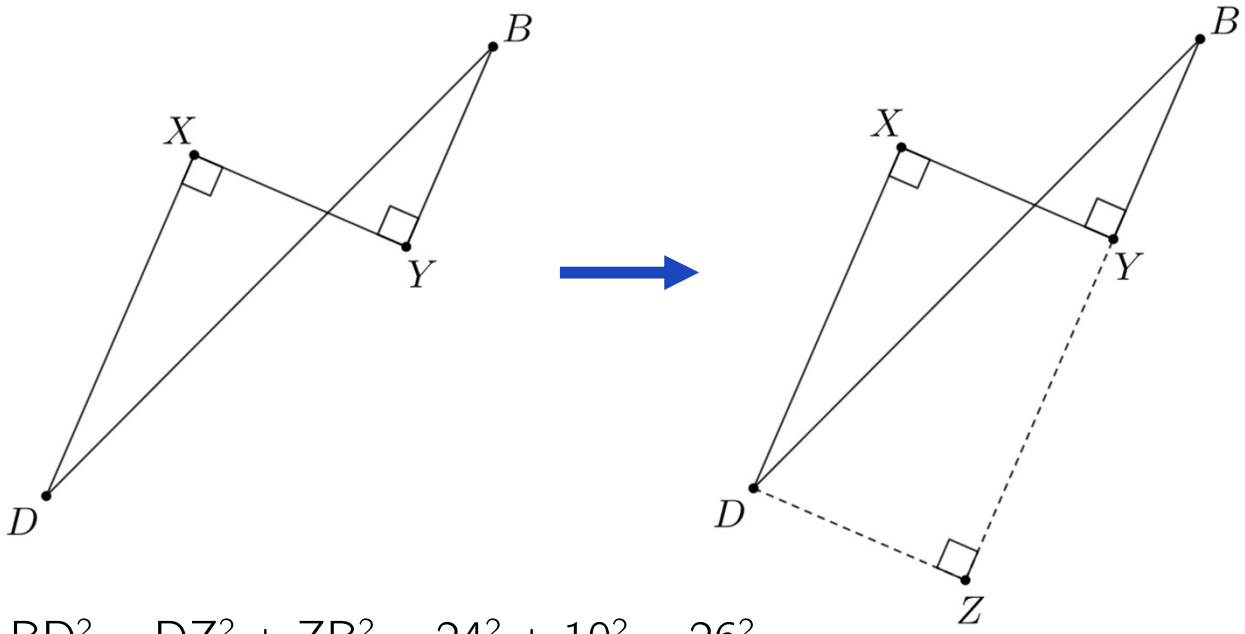
In the figure, ABCD is a square. Let DX = 16, XY = 10 and BY = 8. Find the area of ABCD.

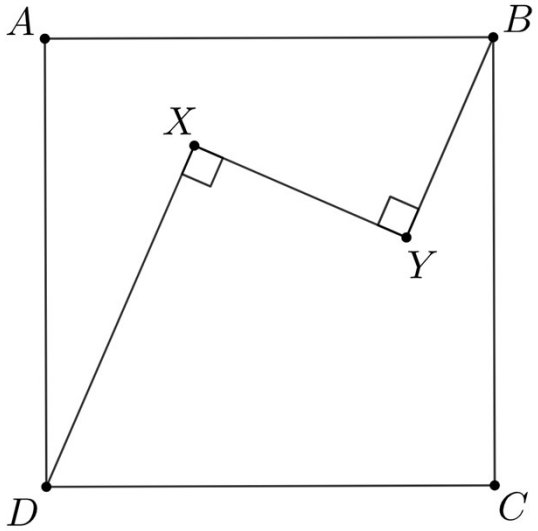


Q4. Zig-zag in Square

Solution

Critical observation: We just need to find BD.





$BD^2 = DZ^2 + ZB^2 = 24^2 + 10^2 = 26^2$.

So, (side-length)$^2 = 13 \times 26 = 338$.

Q5. Ship Patrol

A ship starts at point O. It then sails 1 km east, 2 km north, 3 km west, 4 km south, 5 km east, ... , 10 km north. What is the distance beween the ship and O?

Solution

Horizontal displacement = $|1 - 3 + 5 - 7 + 9| = 5$

Vertical displacement = $|2 - 4 + 8 - 10| = 4$

So, distance^2 = $4^2 + 5^2 = 41$.

Thus, distance = sqrt(41).

Let's have a short break.

We will continue after 5 minutes.

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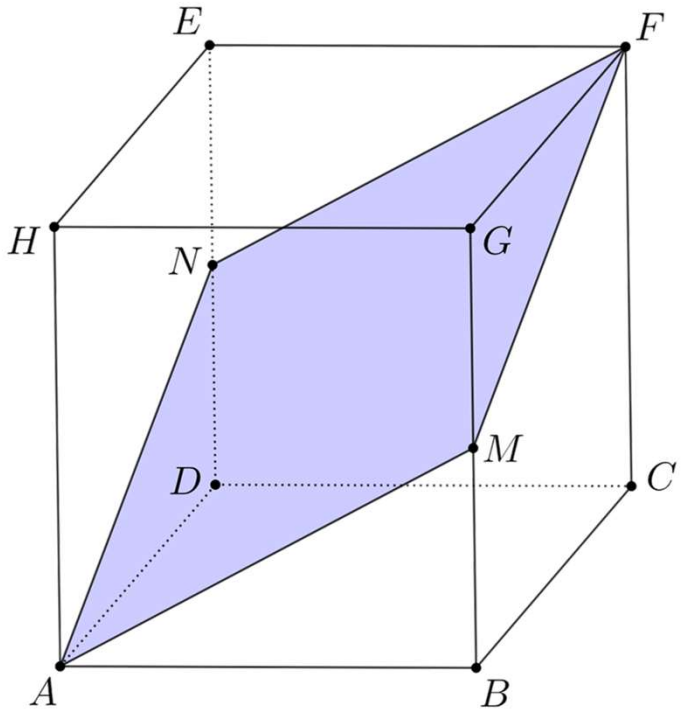
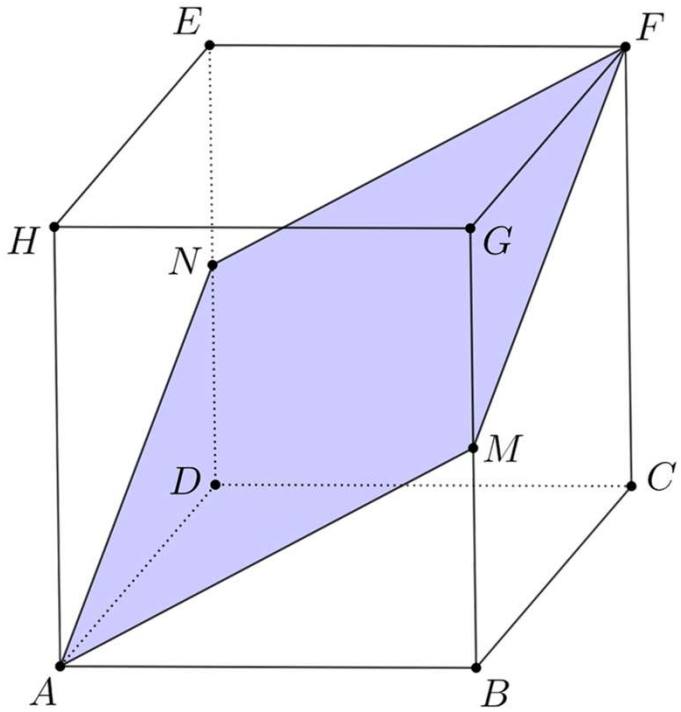
Q6. 3D Pythagoras

In the figure, ABCDEFGH is a cube and M, N are midpoints of BG and DE respectively. Side-length of the cube is 4. What is the area of parallelogram AMFN?



Q6. 3D Pythagoras

In the figure, ABCDEFGH is a cube and M, N are midpoints of BG and DE respectively. Side-length of the cube is 2. What is the area of parallelogram AMFN?



Solution

AMFN is rhombus. So, area = AF $\times$ MN / 2.

$AF^2 = AB^2 + BC^2 + CF^2 = 12$.

$MN^2 = BD^2 = AD^2 + AB^2 = 8$.

Hence, area = sqrt(12 $\times$ 8)/2 = 2sqrt(6).

We both earned our rest.

See you next week.

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