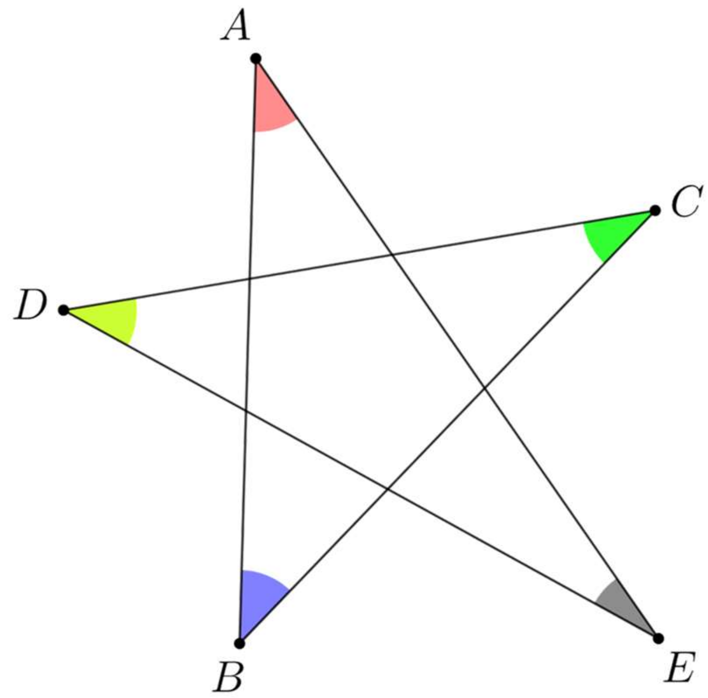
We will begin at 07:03PM

Try this problem in the mean time.

What is the sum of the marked angles in the figure?



Solution to Intro Problem

Solution

$\angle D + \angle E =$ blue angle

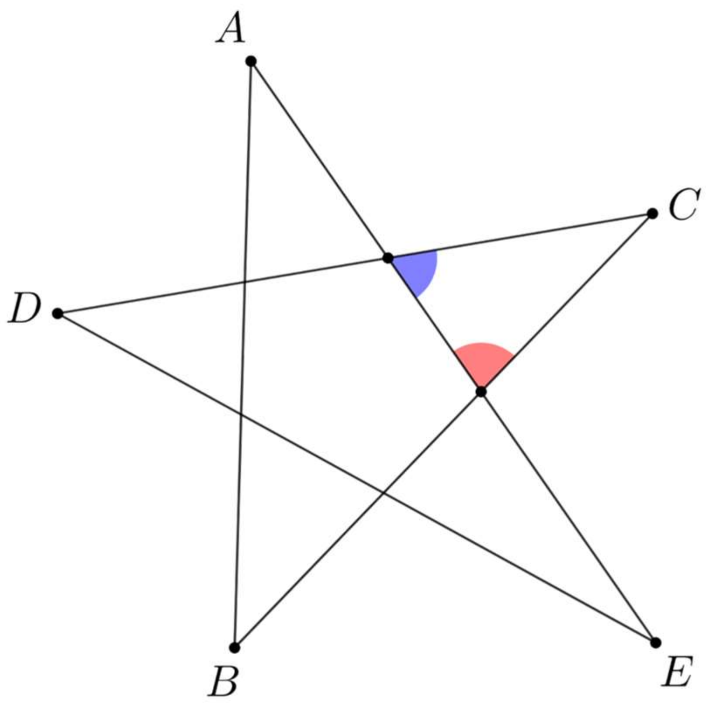
$\angle A + \angle B =$ red angle.

Therefore,

$\angle A + \angle B + \angle C + \angle D + \angle E$

$=$ blue $+$ red $+ \angle C$

$= 180°$



Some Housekeeping

- Homework 1 is due tomorrow. Make sure to do homework. It's the only way to learn actually.
- We have recitation this Thursday 07:00 PM to 08:30 PM as usual for covering homework questions.
- The extra video on this lesson will be posted after the recitation.

Record the meeting.

GEO

I

Lesson – 2

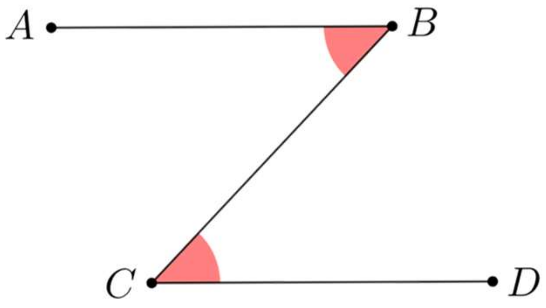
Angle Hunting

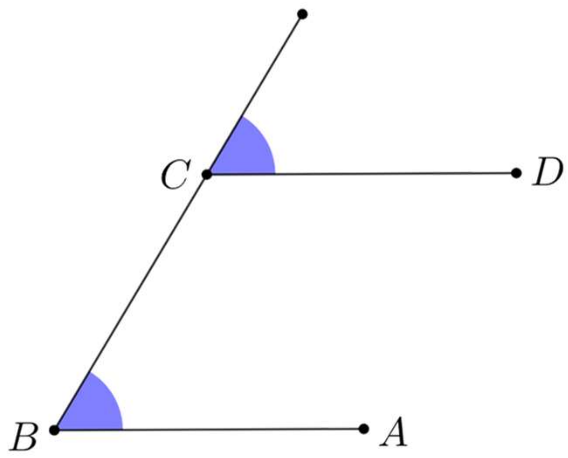
GEO

I

Quick Review

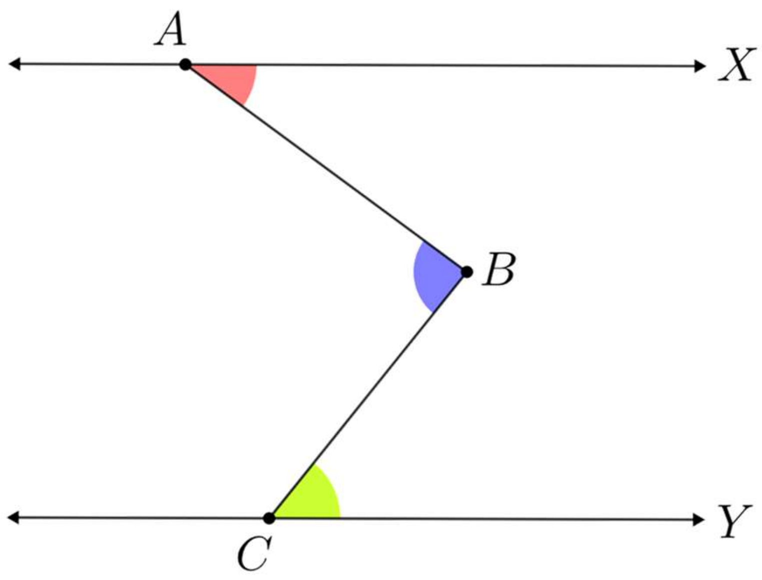
Let AB || CD. Alternate angles are equal, corresponding angles are equal.





Q1. Zig-zag

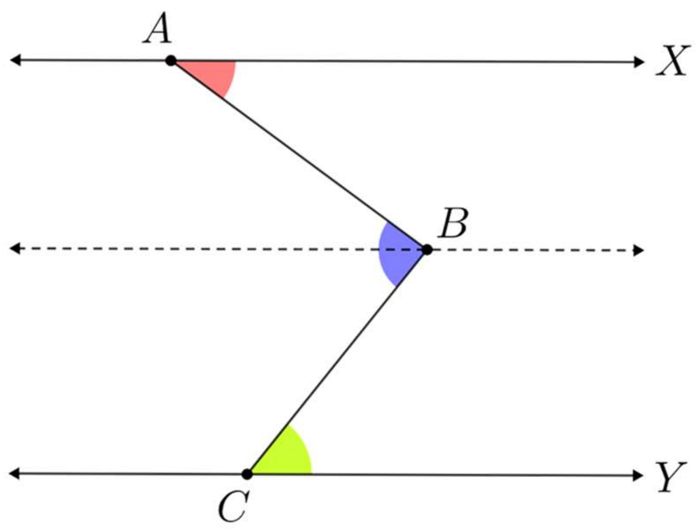
In the picture, AX || CY. $\angle A = 30°$ and $\angle C = 40°$. What is $\angle ABC$?



Q1. Zig-zag

In the picture, AX || CY. ∠A = 30° and ∠C = 40°. What is ∠ABC?

Solution



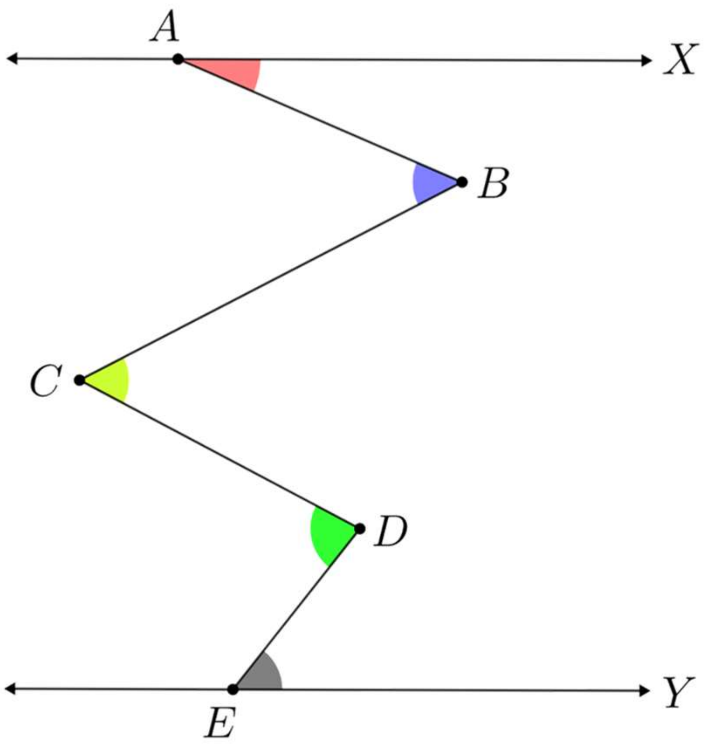
Draw a line parallel to AX and CY through B.

Then, we can see that ∠ABC = ∠XAB + ∠YCB = 70°.

Q2. Zig-zag-zug-zog

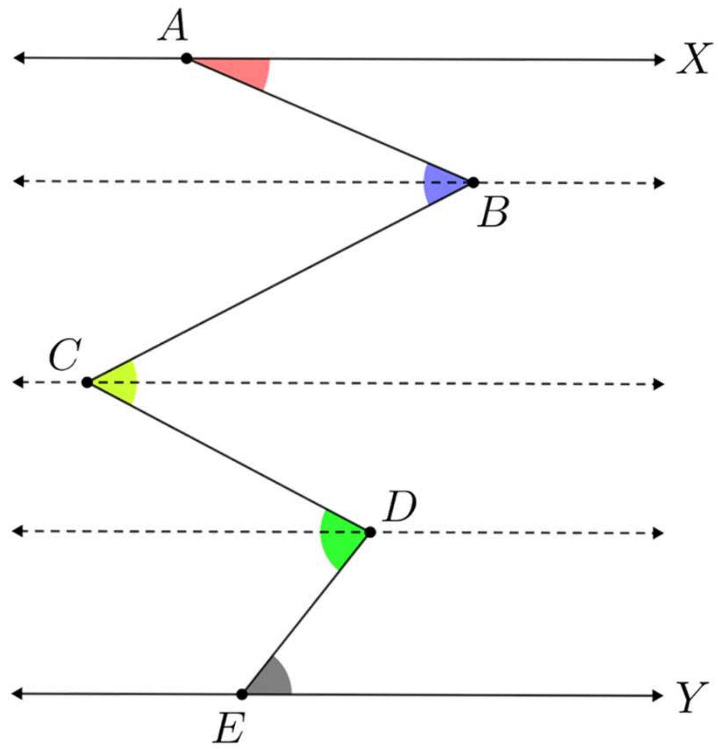
In the picture, AX || EY. Marked angles are given as ∠A = 15°, ∠B = 35°, ∠C = 40° and ∠D = 80°.

Find ∠D.



Q2. Zig-zag-zug-zog

In the picture, AX || EY. Marked angles are given as ∠A = 15°, ∠B = 35°, ∠C = 40° and ∠D = 80°. Find ∠D.



Solution

Draw parallel lines through B, C, D.

Lower part of ∠B = 35° - 15° = 20°.

So, upper part of ∠C = 20°.

Lower part of ∠C = 40° - 20° = 20°.

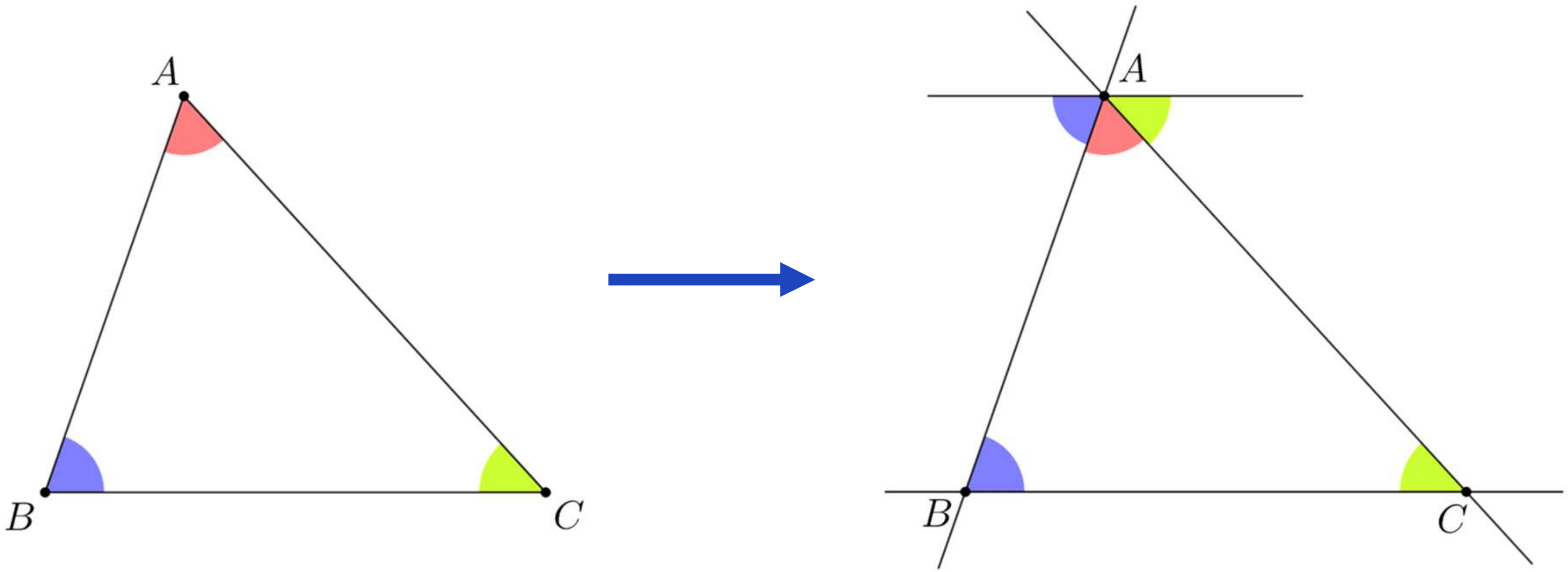
So, upper part of ∠D = 20°.

Lower part of ∠D = 80° - 20° = 60°.

Therefore, ∠E = 60°.

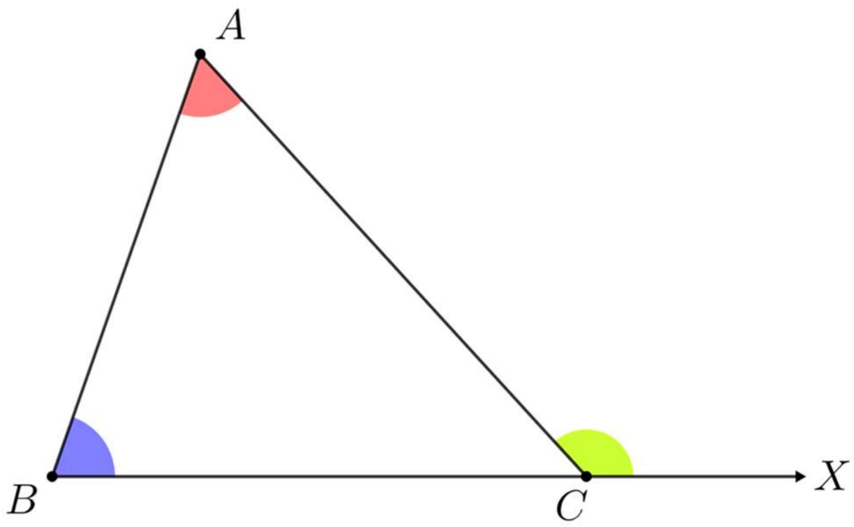
Angles of a Triangle

Theorem: Interior angles of a triangle add up to 180°.



Reason: Move BC "up" through A. By moving angles, we can see that ∠A, ∠B and ∠C add up to 180°.

Corollary: In a triangle, of two interior angles is equal to the exterior angle of the other.



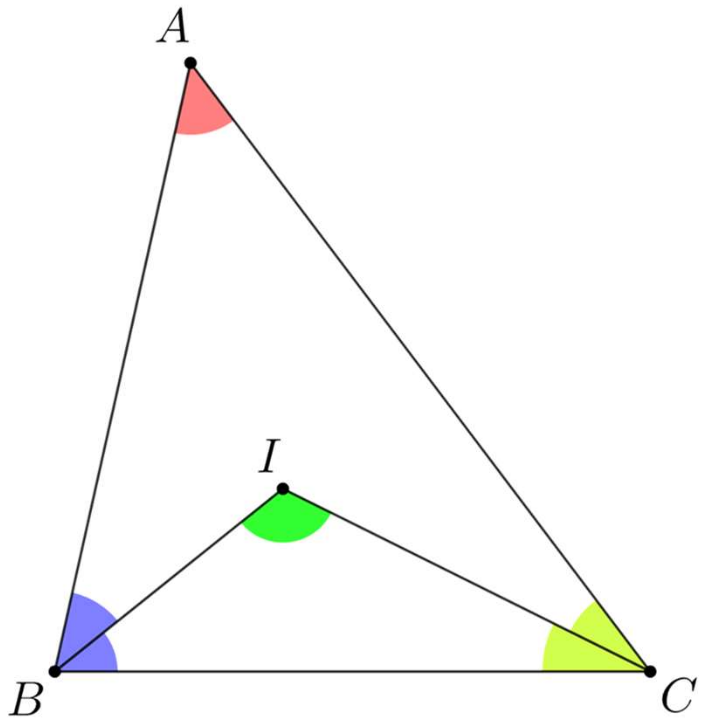
Reason:

Exterior ∠C is complement of interior ∠C. But, ∠A + ∠B is also the complement of interior ∠C.

(Note: x is complement of y if $x + y = 180°$.)

Q3. Angle at the Incenter

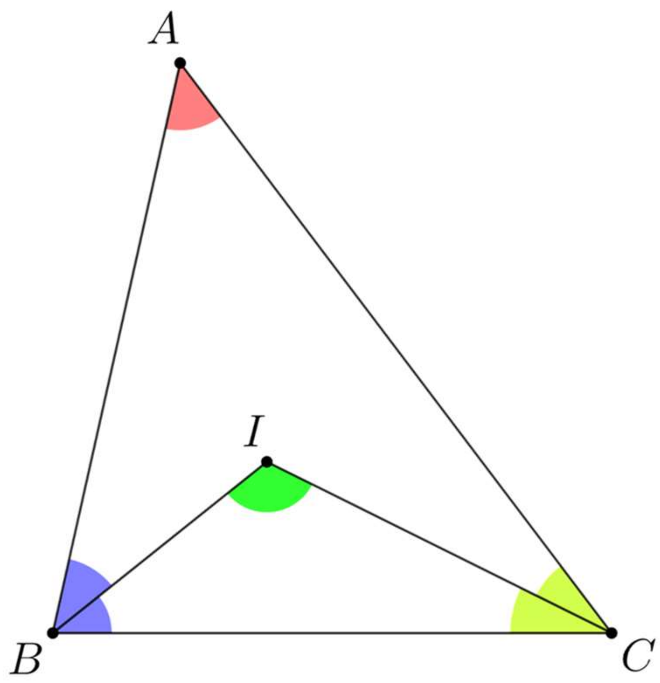
In triangle ABC, BI and CI are interior angle bisectors. If $\angle A = 40°$, what is $\angle BIC$?



Q3. Angle at the Incenter

In triangle ABC, BI and CI are interior angle bisectors. If ∠A = 40°, what is ∠BIC?

Solution



To find ∠BIC, we need blue + yellow.

But, 2 × blue + 2 × yellow = 180° - 40° = 140°.

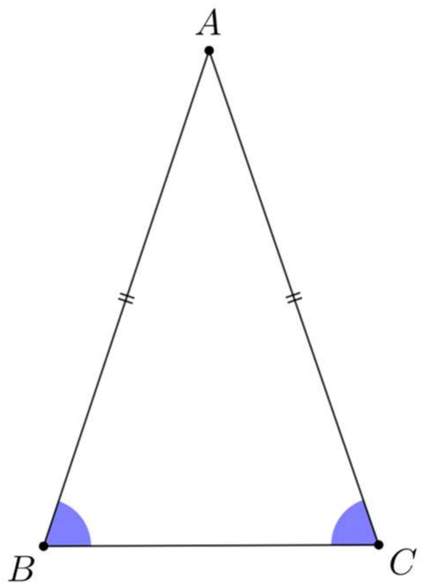
Therefore, blue + yellow is 70°.

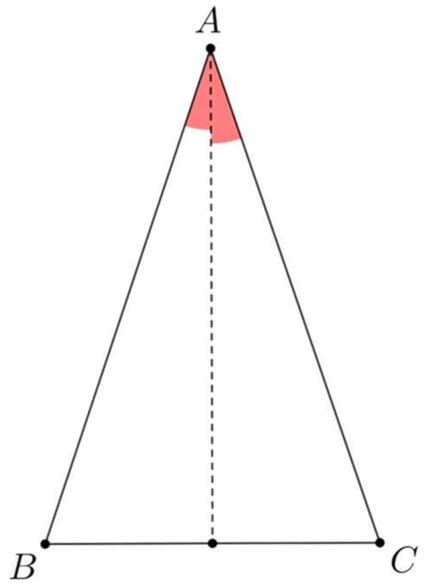
Hence, ∠BIC = 180° - 70° = 110°.

General Relation: ∠BIC = 90° + ∠A/2

Angles of an Isosceles Triangle

Theorem: A triangle is isosceles if and only if its base angles are equal.

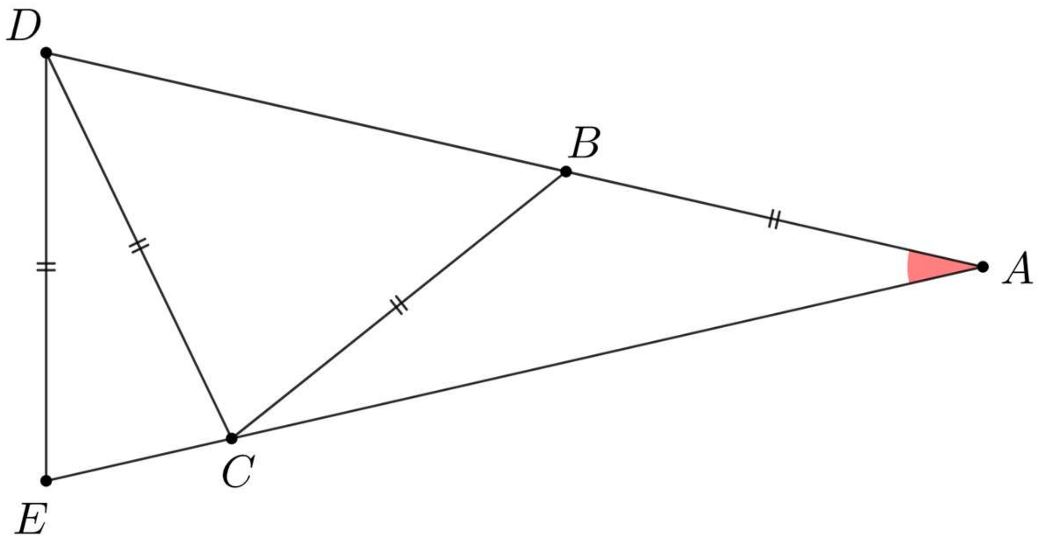




Reason: Consider the bisector of the "top" vertex. A triangle is isosceles if and only if the resulting triangles are congruent. Also, base angles are equal if and only if the resulting triangles are congruent.

Q4. Chain Isosceles

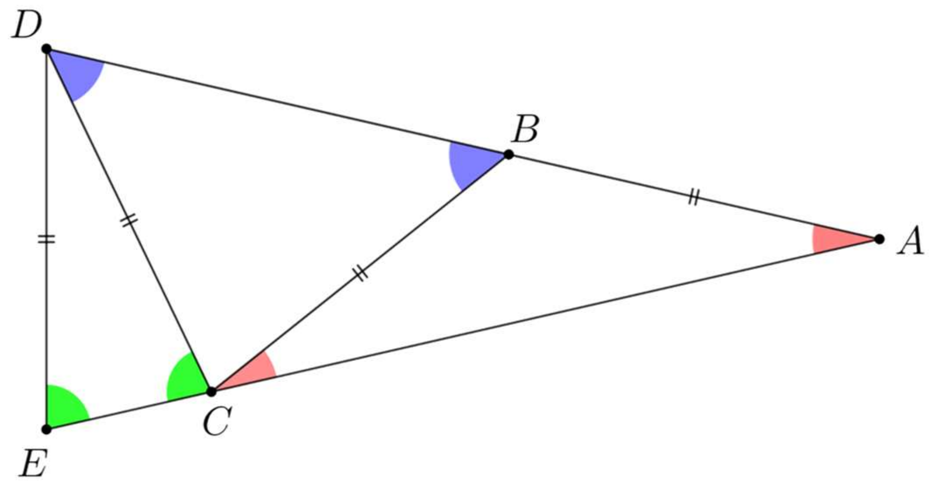
In the figure, AB = BC = CD = DE and AD = AE. What is ∠A?



Q4. Chain Isosceles

In the figure, AB = BC = CD = DE and AD = AE. What is ∠A?

Solution



Angles of the same colour are equal.

Looking at ABC, blue = 2 × red.

Looking at ACD, green = blue + red = 3 × red.

But from AD = AE, green + green + red = 180°.

So, 7 × red = 180° and thus ∠A = 180°/7.

Let’s have a short break.

We will continue after 5 minutes.

GEO

I

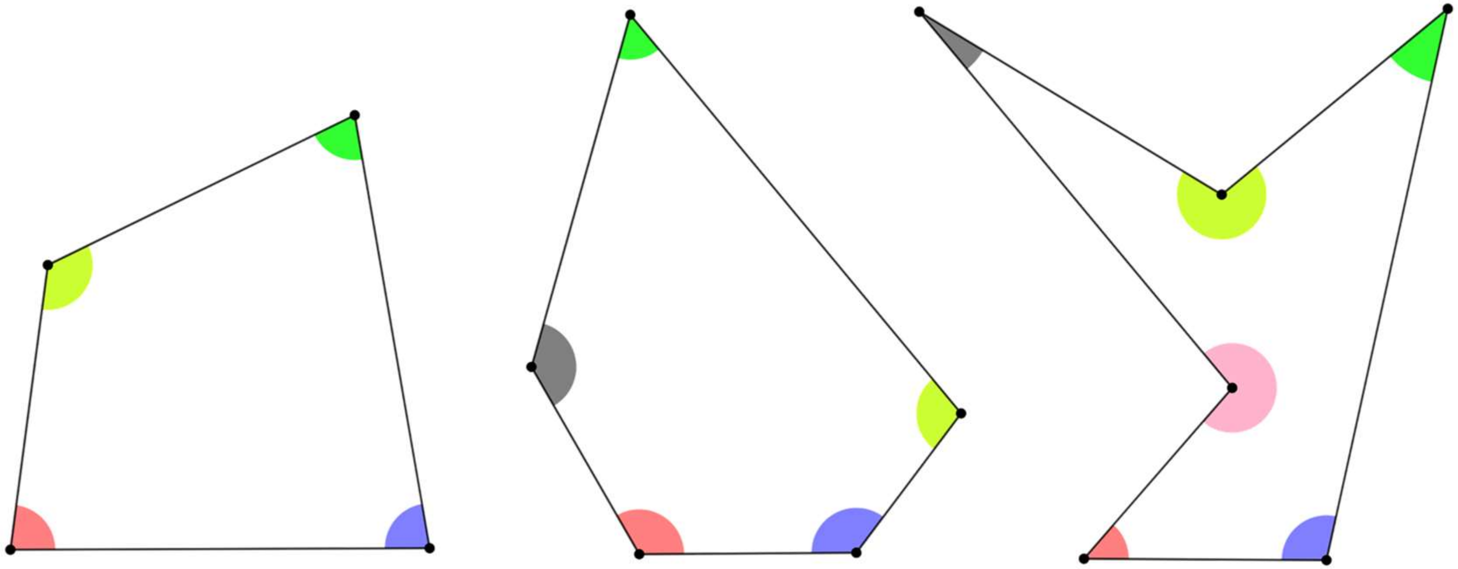
Record the meeting.

GEO

I

Interior Angle Sum

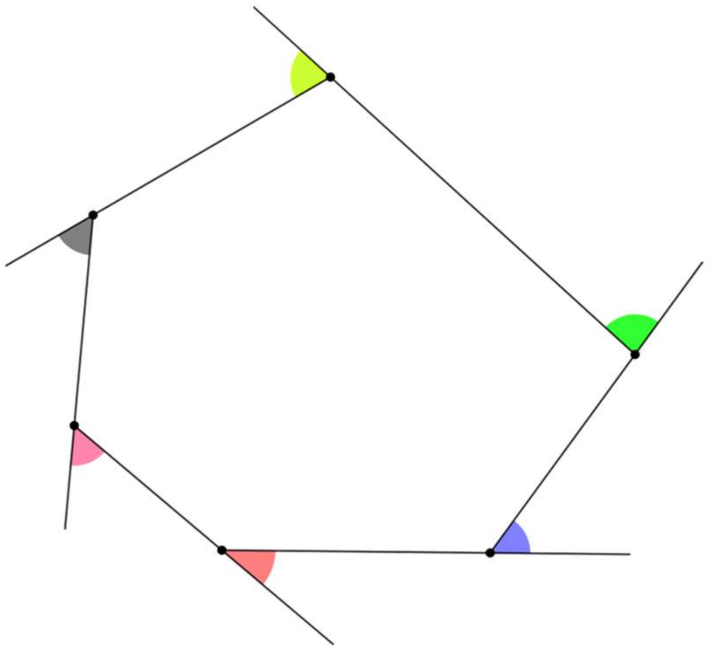
Theorem: Interior angles of an n-gon (not necessarily regular) add up to $n-2$ straight angles.



Reason: An n-gon can be decomposed into $n-2$ triangles (not easy to prove). Interior angle sum of the n-gon is equal to interior angle sum of those triangles.

Exterior Angle Sum (Walking-around trick)

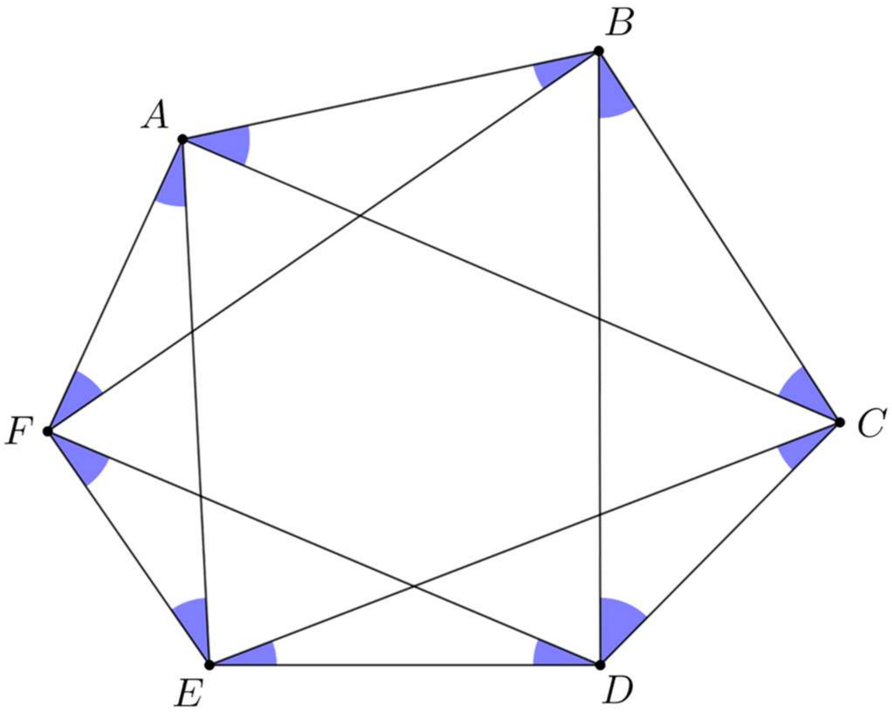
Theorem: Exterior angles of a convex n-gon (not necessarily regular) add up to 360°.



Reason: Imagine walking around the n-gon starting inside some segment. Then, exterior angle sum is just the total angle you rotated after one walk around the n-gon. So, exterior angle sum is 360°.

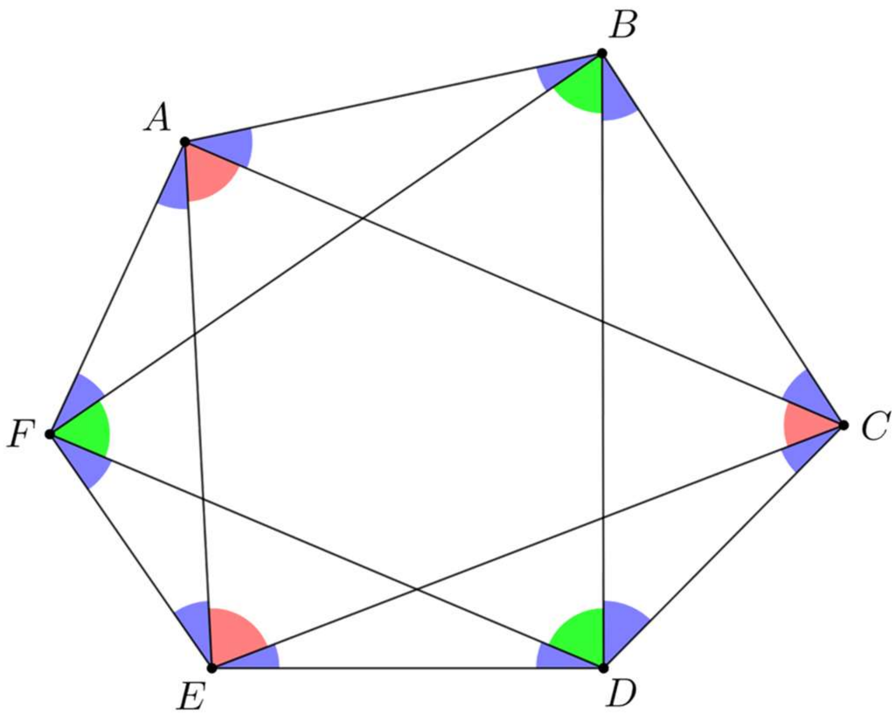
Q5. Blue Angles in Hexagon

What is the sum of all the blue angles in the following figure?



Q5. Blue Angles in Hexagon

What is the sum of all the blue angles in the following figure?



Solution

Red angle sum = 180°.

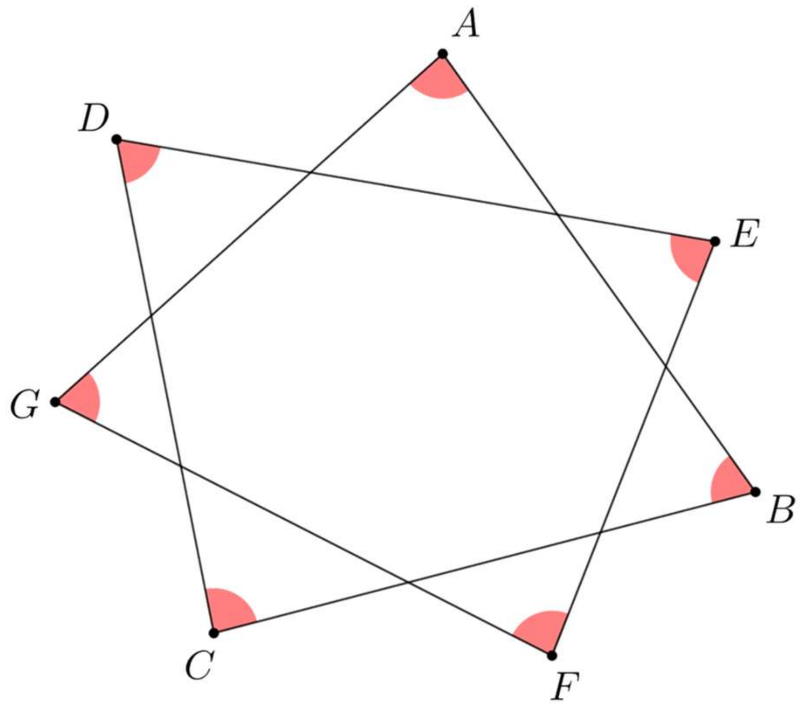
Green angle sum = 180°.

Red, green, blue angle sum = 4 × 180° = 720°.

So, blue angle sum = 720° - 180° - 180° = 360°.

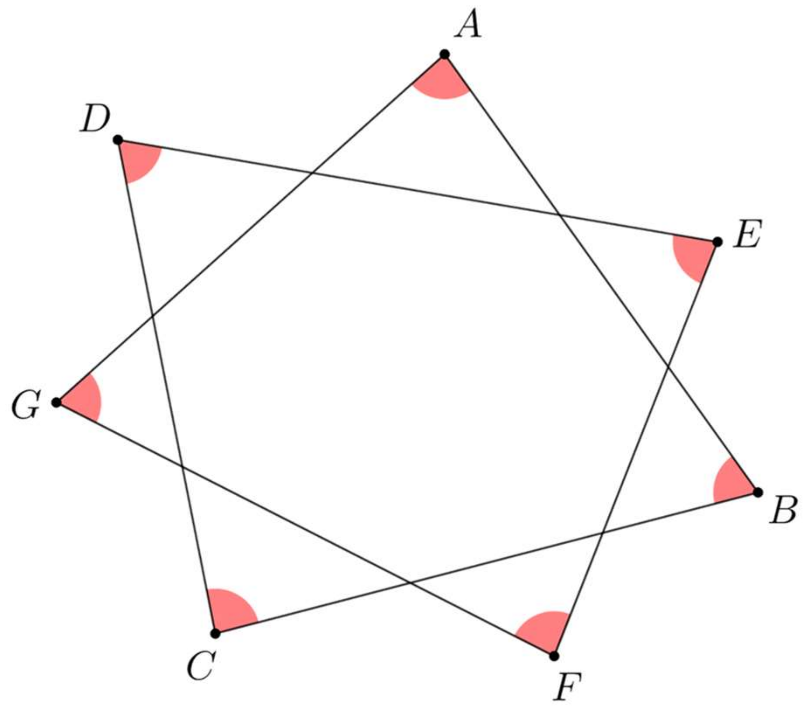
Q6. Red Angles in a Heptagonal Star

What is the sum of all the red angles in the following figure?



Q6. Red Angles in a Heptagonal Star

What is the sum of all the red angles in the following figure?



Solution

By walking around the polygon trick, exterior angle sum is $2 \times 360° = 720°$.

So, sum of (180° - a red angle) = 720°.

So, $7 \times 180°$ - (sum of red angles) = 720°.

Therefore, sum of red angles = 540°.

I guess we both earned our rest.

See you soon!

GEO

I