Lesson – 1

Area Hunting

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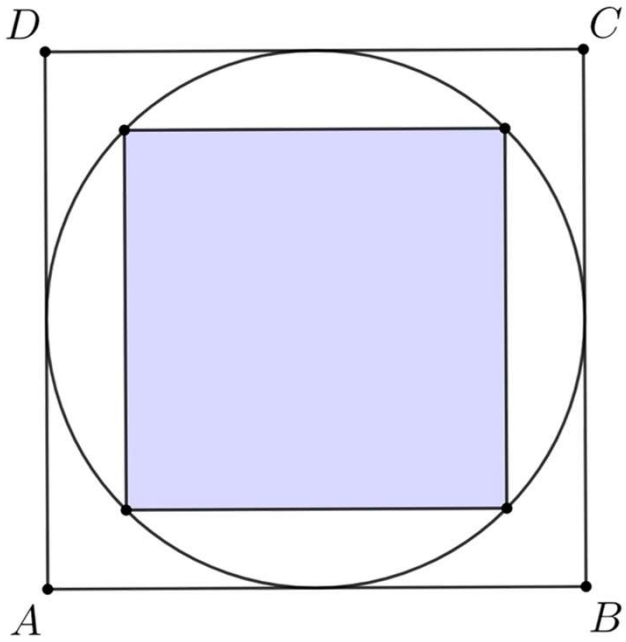
We will start at 07:03 PM

Try this question in the mean time:

In the figure, a big square circumscribes a circle and a small square is inscribed in the circle. The area of the small square is 10. What is the area of the big square?



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Who am I?

Name: Hein Thant Aung

Contact: Facebook Messenger (link in course outline)

Email (heinta2002@gmail.com)

I was a contestant of IMO2017 representing Myanmar. I've taught olympiads since then. Now, I'm studying mathematics at a university in Hong Kong.

About this Course

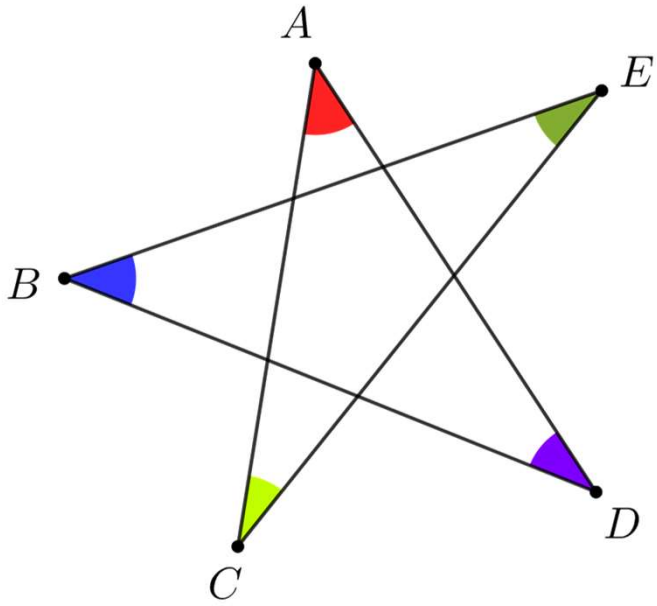
- Everything you need to know is in google classroom.
- Schedule: We will have lessons on Tuesdays weekly and recitations on Thursdays biweekly. We will have a test in the end. All sessions are from 07:00 PM to 08:30 PM.

- Grading: Weekly homework makes up 60% of your final grade and the final test makes up 40%.

- Doing homework is the only way to learn the material. So, please make sure to take it seriously.

Content

We will have 5 lessons and 4 videos. The tentative content are as follows:



Lesson 1: Area hunting

Lesson 2: Angle hunting

Lesson 3: Right triangles and Pythagoras

Lesson 4: Ratios and proportions

Lesson 5: Congruences and symmetry

Videos are for backing up more challenging problems and more advanced techniques.

Lesson – 1

Area Hunting

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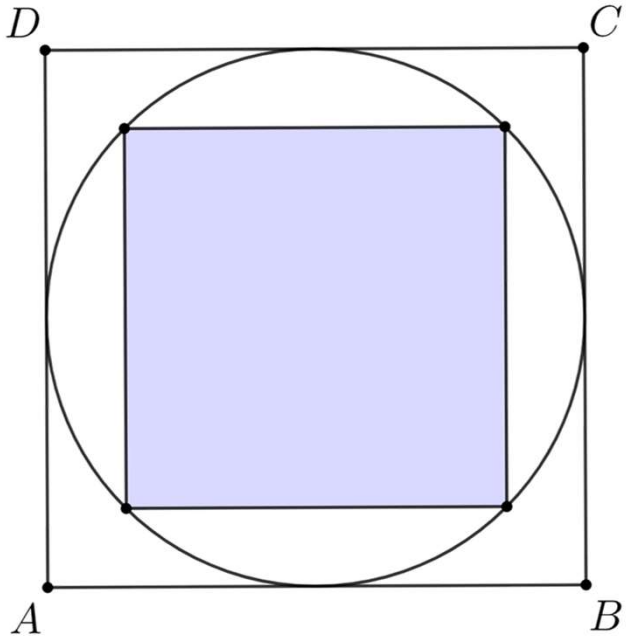
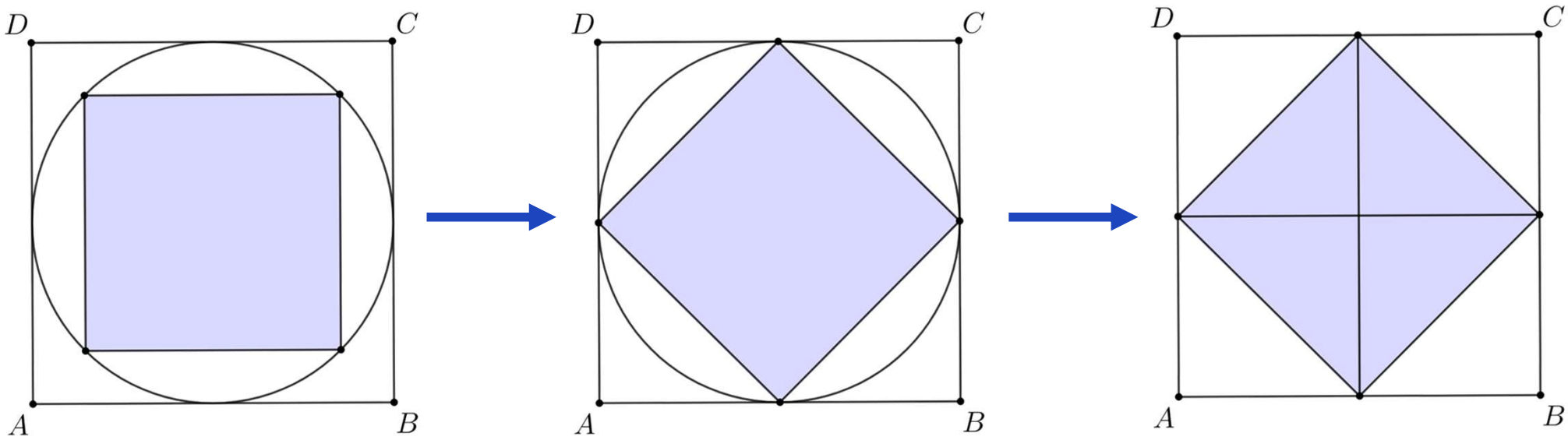
Q1. The First Taste

In the figure, a big square circumscribes a circle and a small square is inscribed in the circle. The area of the small square is 10. What is the area of the big square?



Q1. The First Taste



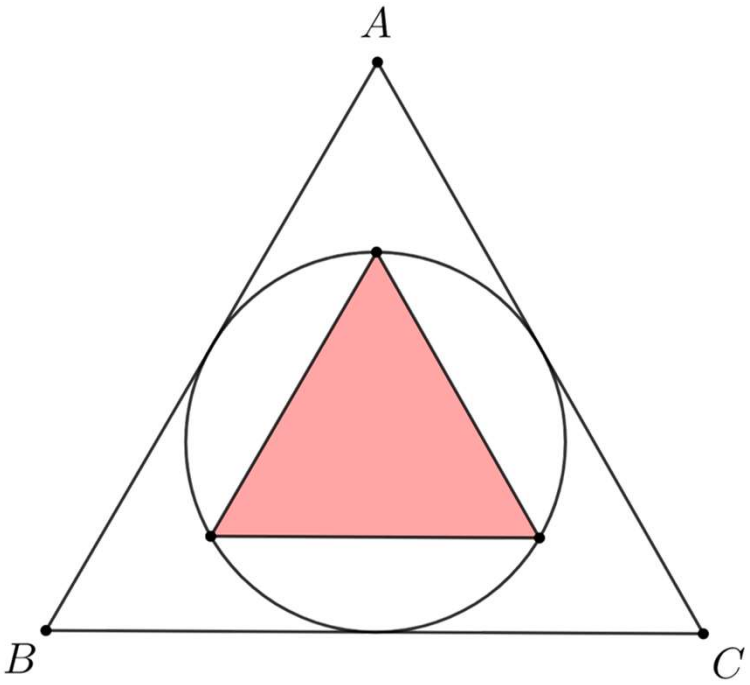
Area does not change under rotation.

So, the picture tells us that the big square is 2 times bigger (in area) than the small square.

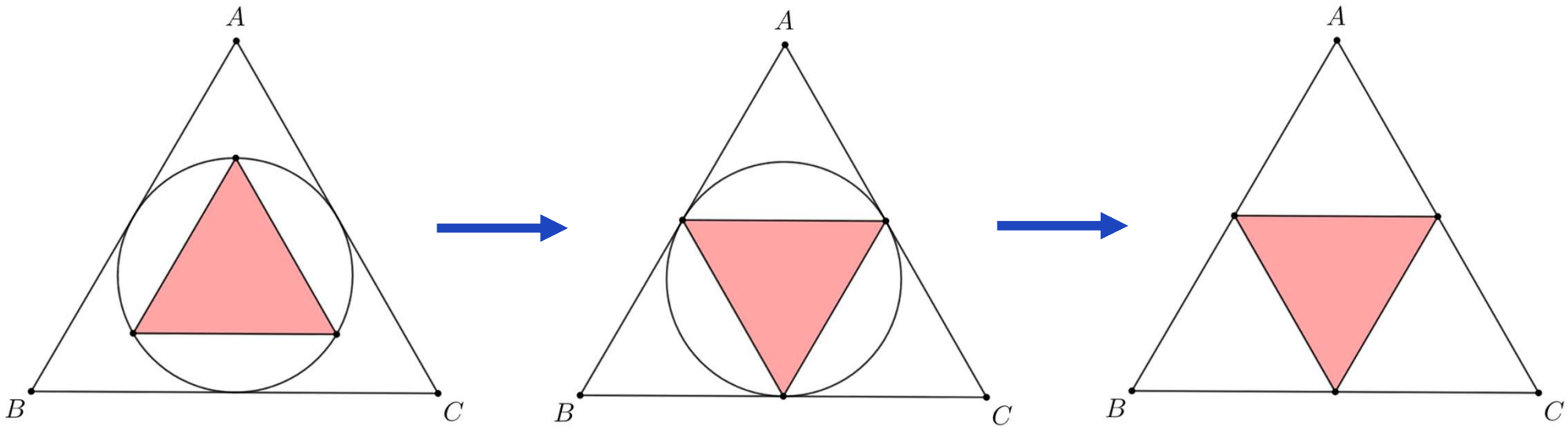
Therefore, the area of big square is $2 \times 10 = 20$.

Q2. What about a Triangle?

In the figure, a big equilateral triangle circumscribes a circle and a small equilateral triangle is inscribed in the circle. The area of the small equilateral triangle is 10. What is the area of the big equilateral triangle?



Q2. What about a Triangle?



Area does not change under rotation.

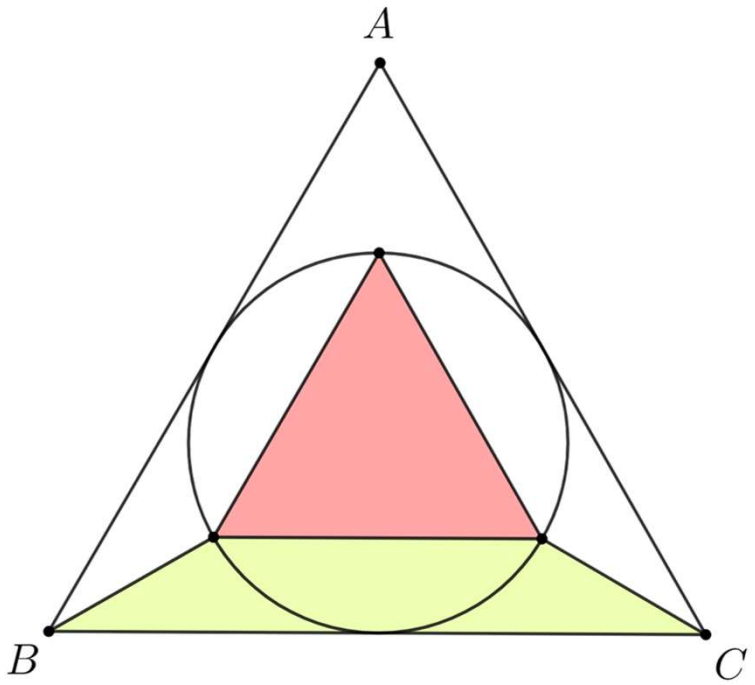
So, the picture tells us that the big triangle is 4 times bigger (in area) than the small triangle.

Therefore, the area of big triangle is $4 \times 10 = 40$.

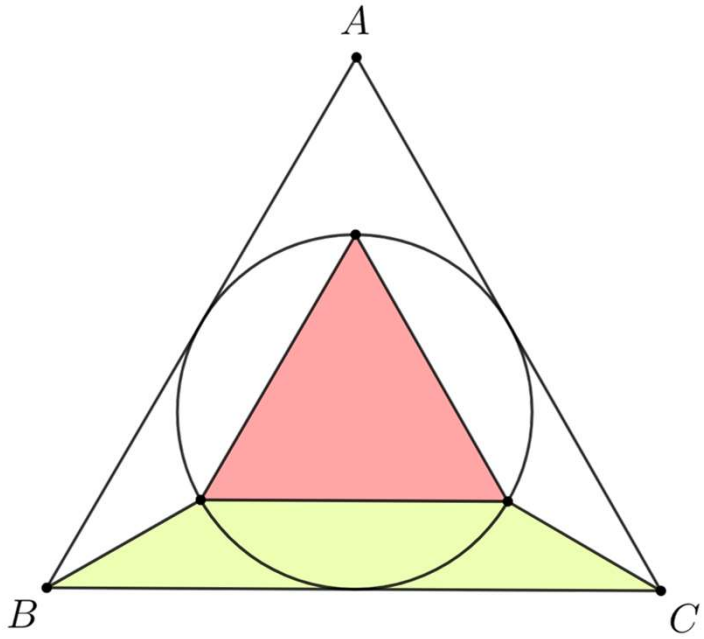
Q3. Follow Up

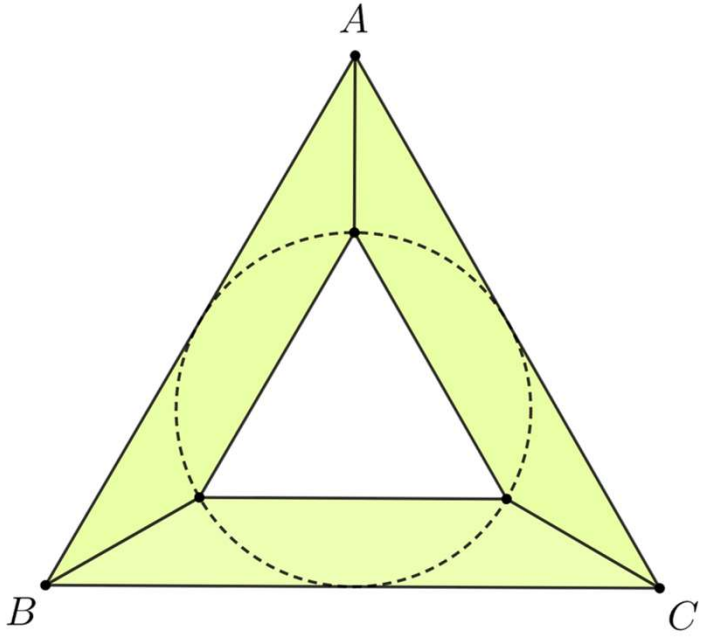
What about now? Area of the small equilateral triangle is 10.

What is the yellow area?



Q3. Follow Up





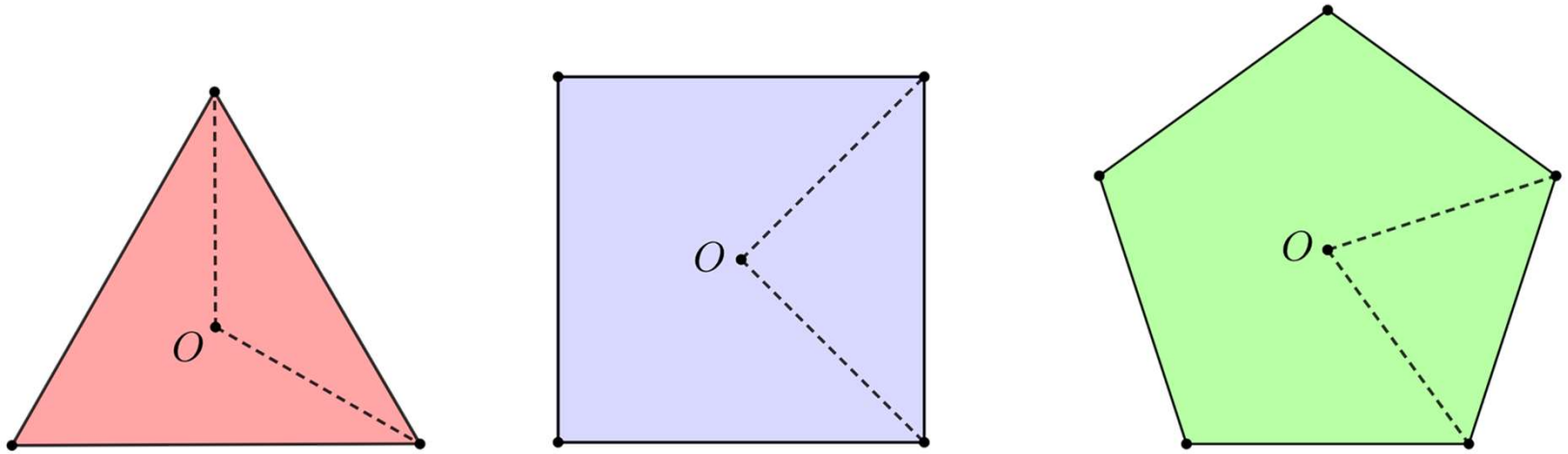
Area of triangle ABC is $4 \times 10 = 40$.

So, area of three yellow regions (trapeziums) is $40 - 10 = 30$.

Hence, area of each yellow region is also 10.

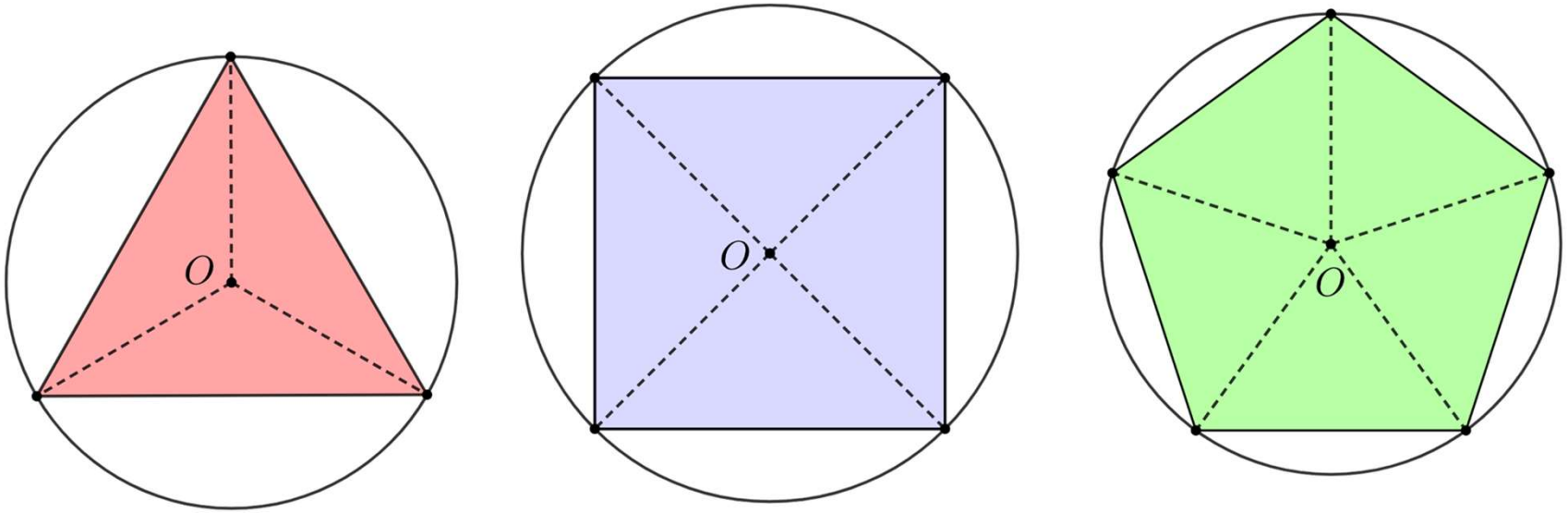
Symmetry of Regular Polygons

An n-sided regular polygon has a point O inside it such that when you rotate the polygon by $360/n$ degrees, the resulting polygon coincides with the original one.



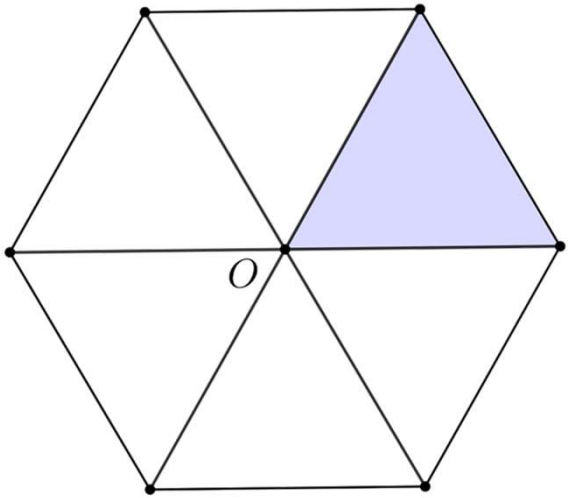
Symmetry of Regular Polygons

There is a circle through the vertices of a regular polygon is called the circumcircle. The point O is also the centre of the circumcircle.



Symmetry of Regular Hexagon

Symmetry in regular hexagons is particularly important.



- The centre O forms an equilateral triangle with each side.
- The diagonals joining opposite vertices concur at O. (Also true for any even regular polygon)

Let's have a short break!

We will continue in 5 minutes.

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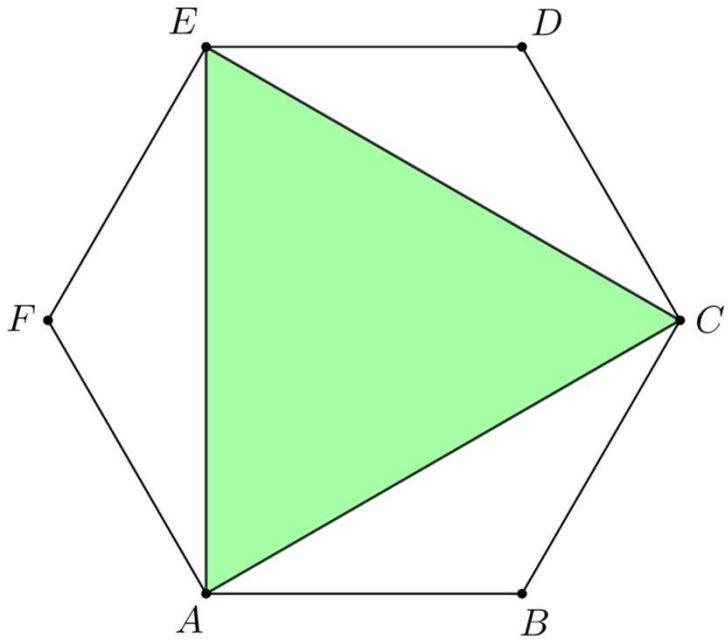
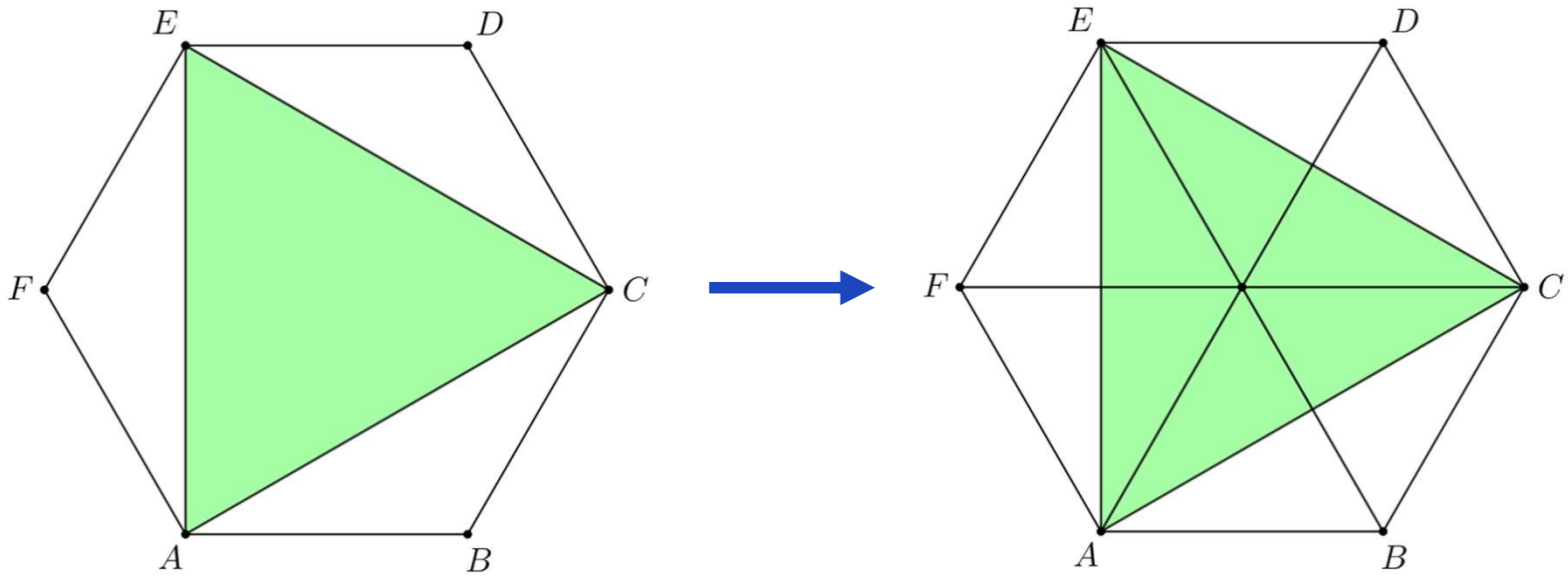
Q4. Hexagon Area I

In the picture, area of ABCDEF is 30. What is the area of triangle ACE?



Q4. Hexagon Area I



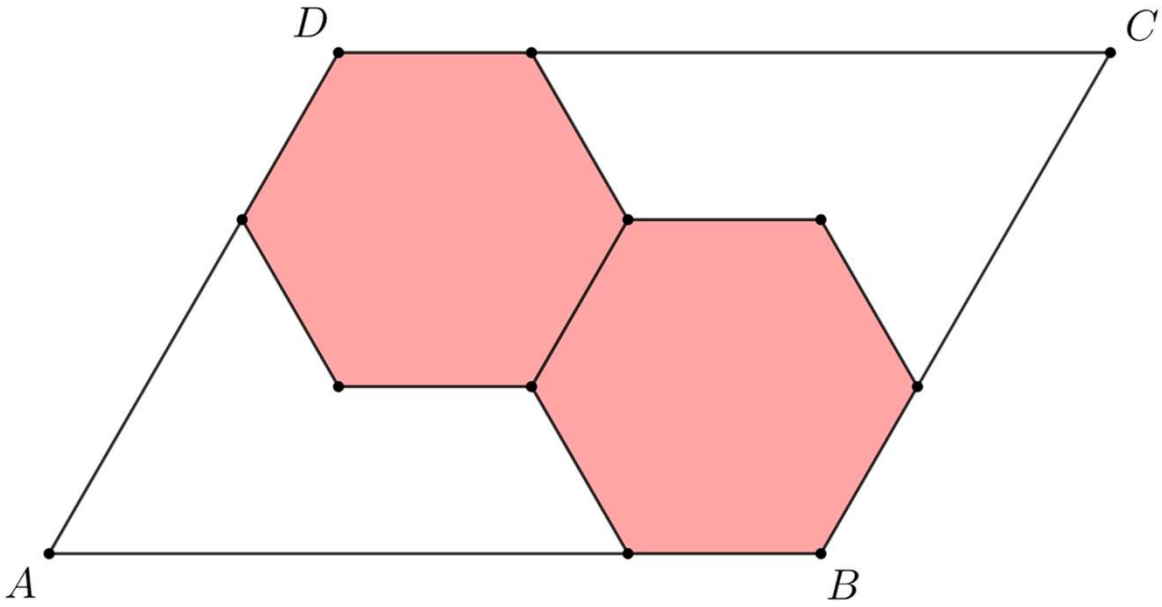
Diagonals of a parallelogram divide it into two congruent triangles.

Therefore, area of ACE is half of ABCDEF.

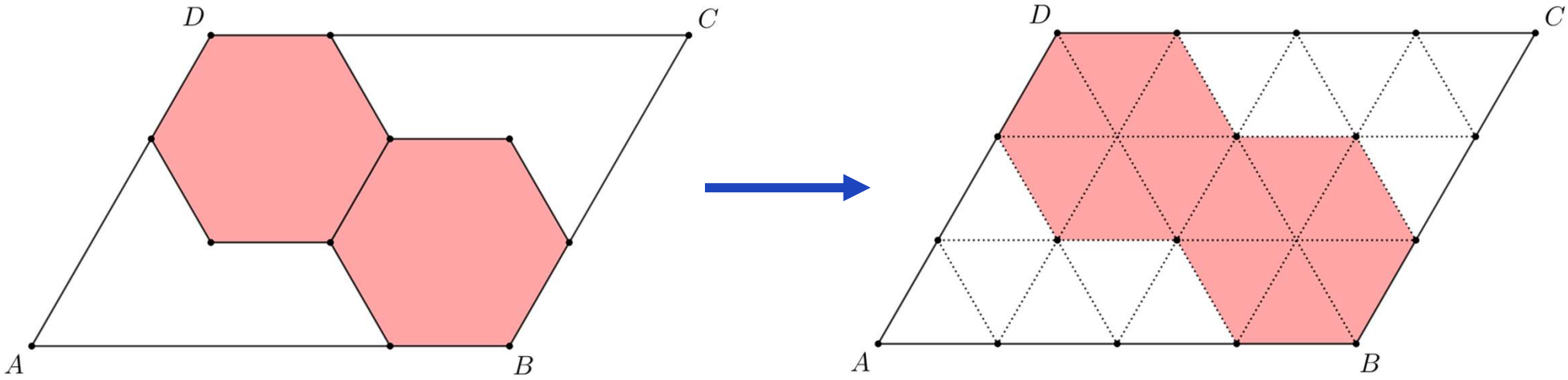
So, area of ACE is equal to half of $30 = 15$.

Q5. Hexagon Area II

In the picture, area of the big parallelogram ABCD is 120. The red area is made up with two congruent regular hexagons. What is the red area?



Q5. Hexagon Area II



ABCD is made up with 24 small equilateral triangles.

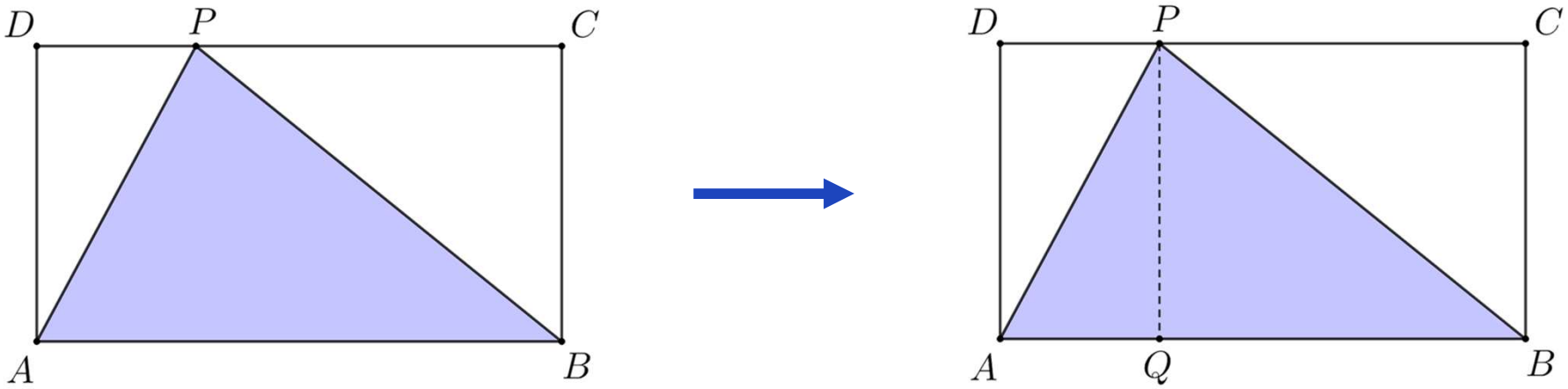
The red area is made up with 12 small equilateral triangles.

Thus, red area is half of ABCD.

Hence, red area is equal to half of $120 = 60$.

What is the Area of a Triangle?

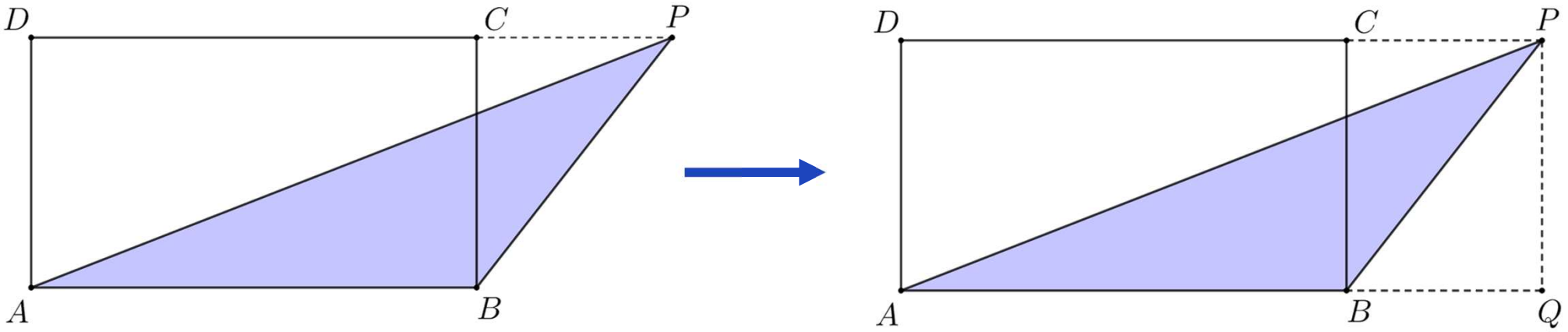
How much area does triangle ABP take up inside rectangle ABCD?



- [APQ] is half of [ADPQ].
- [BPQ] is half of [BCPQ].
- Therefore, [ABP] is half of [ABCD].

What is the Area of a Triangle?

What if P lies outside segment CD, but still on line CD?

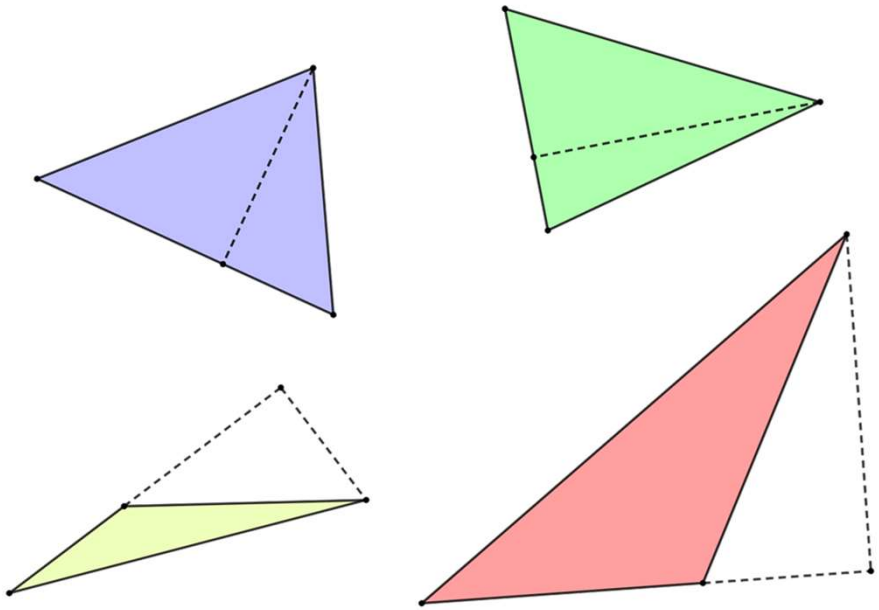


- [APQ] is half of [ADPQ].
- [BPQ] is half of [BCPQ].
- Therefore, [ABP] is half of [ABCD].

What is the Area of a Triangle?

Area of a triangle is equal to

$$\text{Half} \times \underbrace{\text{Length of one side}}_{\text{base}} \times \underbrace{\text{Distance from the other vertex to that side}}_{\text{height}}$$

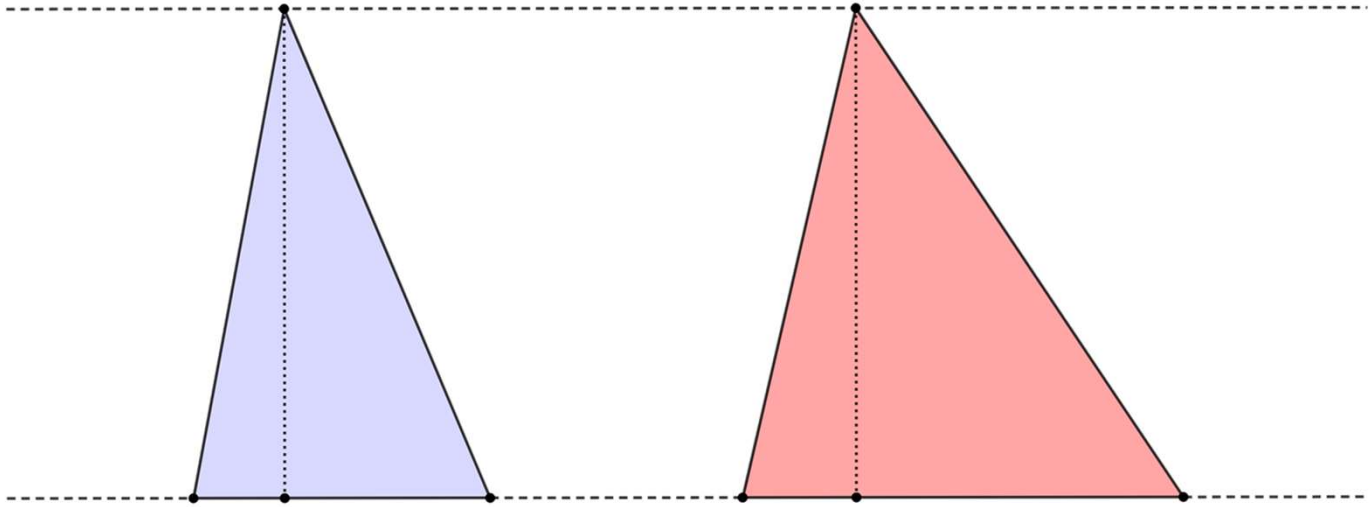


Triangles with Same Height

If two triangles have the same height, then

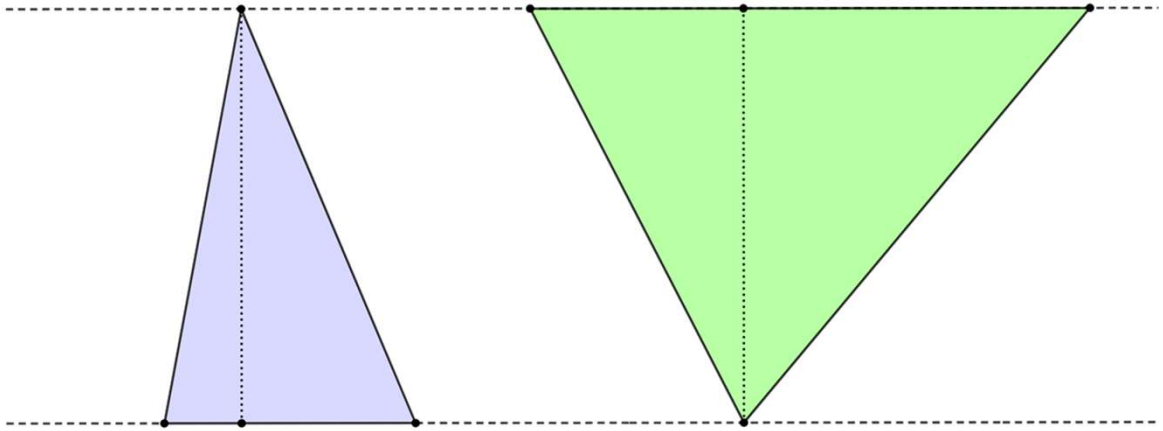
Ratio of areas = Ratio of bases.

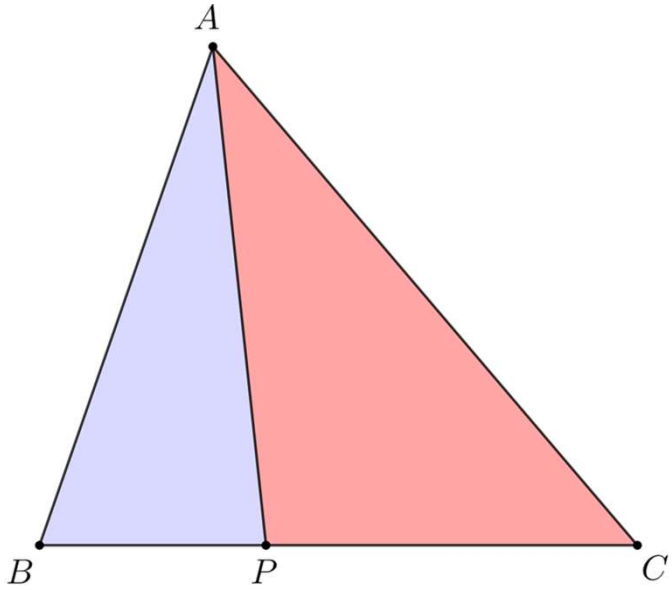
TIP: This is mainly used to convert side-length information into area information.



Triangles with Same Height

Here are examples of triangles with same height:





IMPORTANT: Medians divide the triangle into two equal-area triangles.

Q6. Quadrilateral into 4 parts

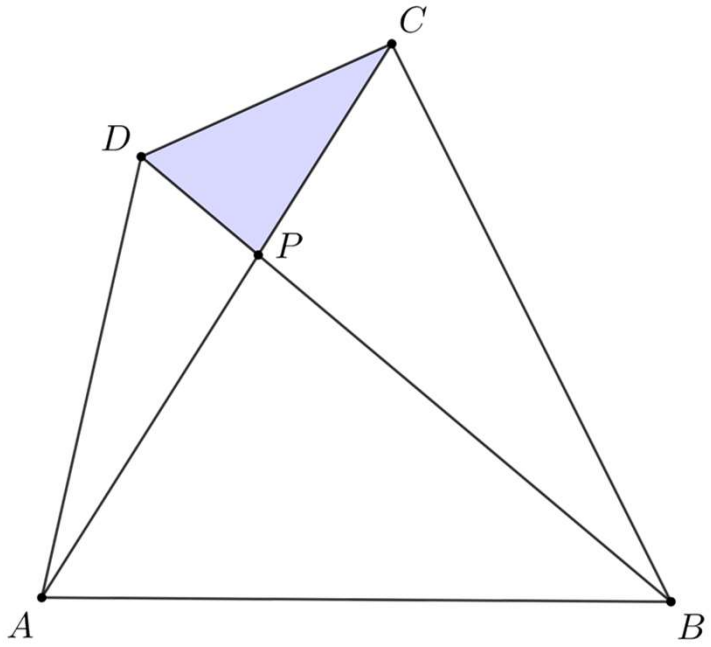
Diagonals AC and BD of quadrilateral ABCD meet at P.

Suppose that AP : PC = 2 : 1 and BP : PD = 3 : 1.

Let the area of PCD be 6.

(a) What is the area of PAD?

(b) What is the area of ABCD?



Q6. Quadrilateral into 4 parts

Solution

Area of PAD is 2 times area of PCD.

So, area of PAD is 12.

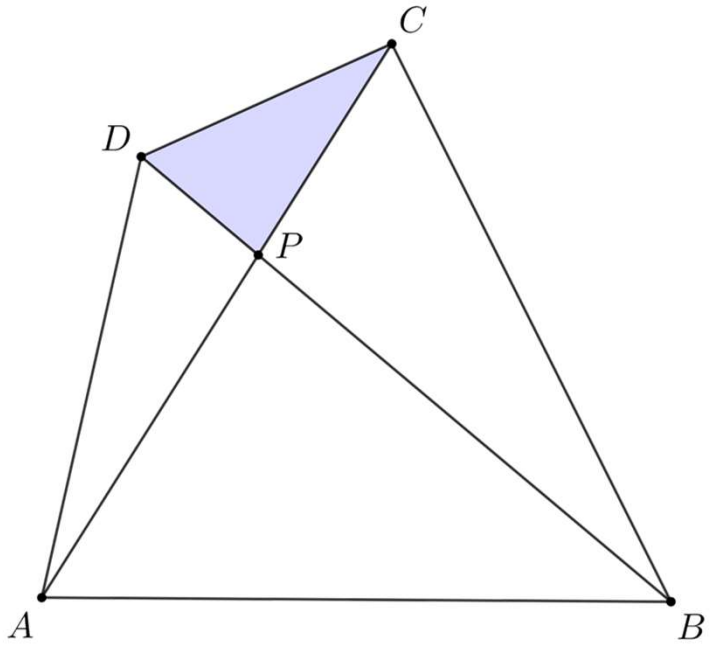
Area of PBC is 3 times area of PCD.

So, area of PBC is 18.

Area of PAB is 3 times area of PAD.

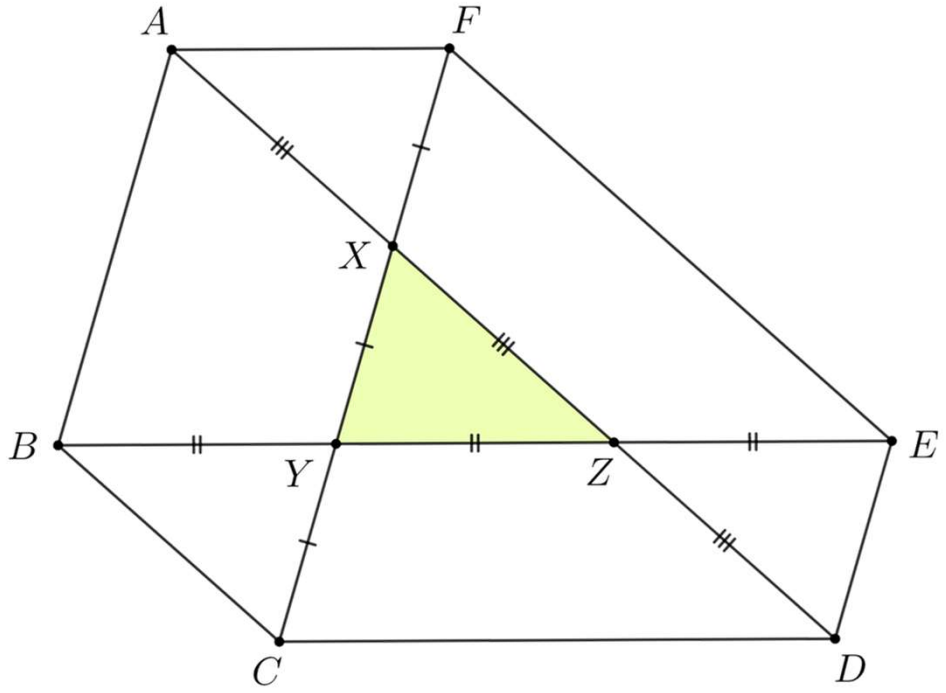
So, area of PAB is 36.

Hence, area of ABCD is $6 + 12 + 18 + 36 = 72$.

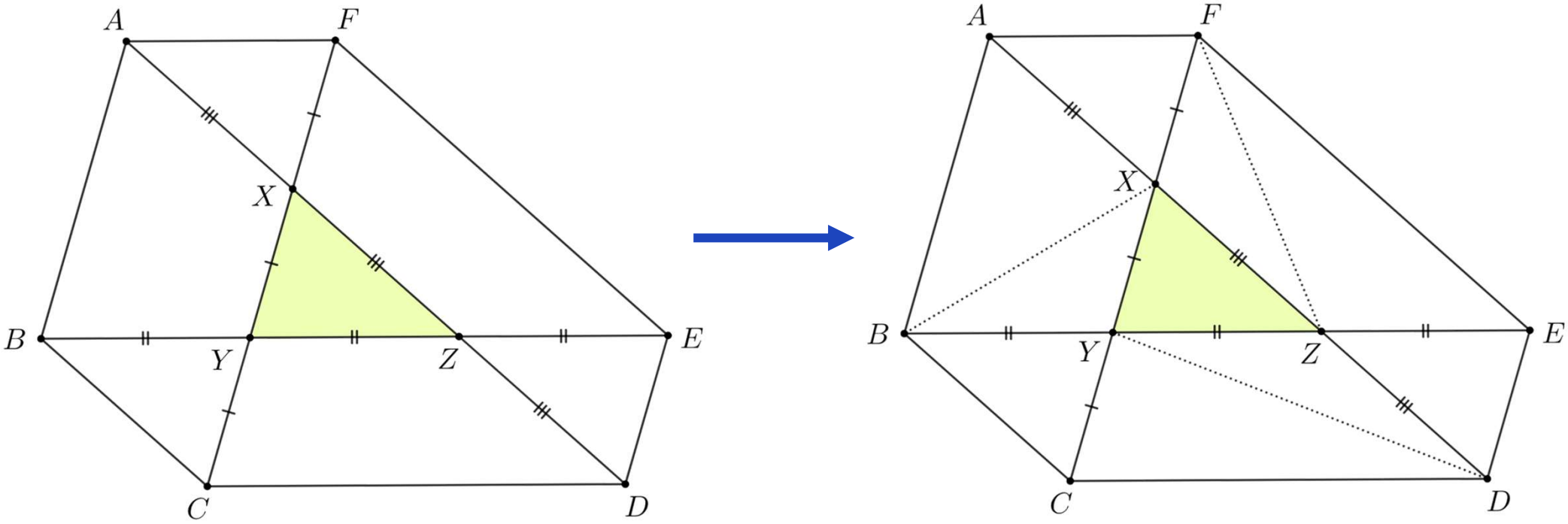


Q7. Little Triangle in Big Hexagon

In the figure, PX = XY = YC, BY = YZ = ZE and AX = XZ = ZD. Suppose that XYZ has area 3. What is the area of ABCDEF?



Q7. Little Triangle in Big Hexagon



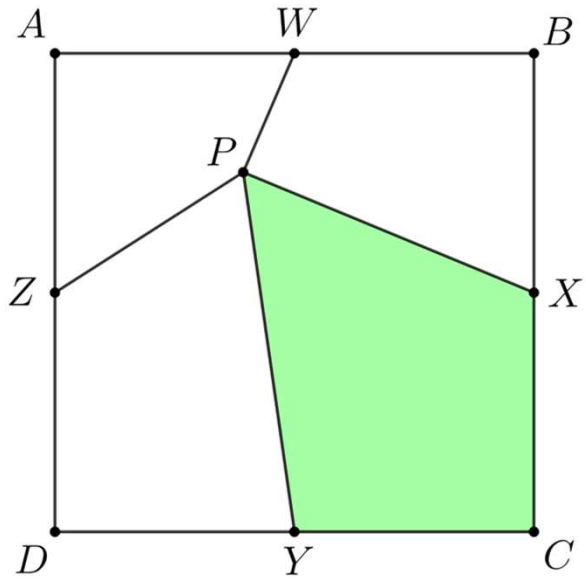
XYB has same area with XYZ. So, XYB is also area 3.

ABX has same area with BXZ. So, ABX has area 6.

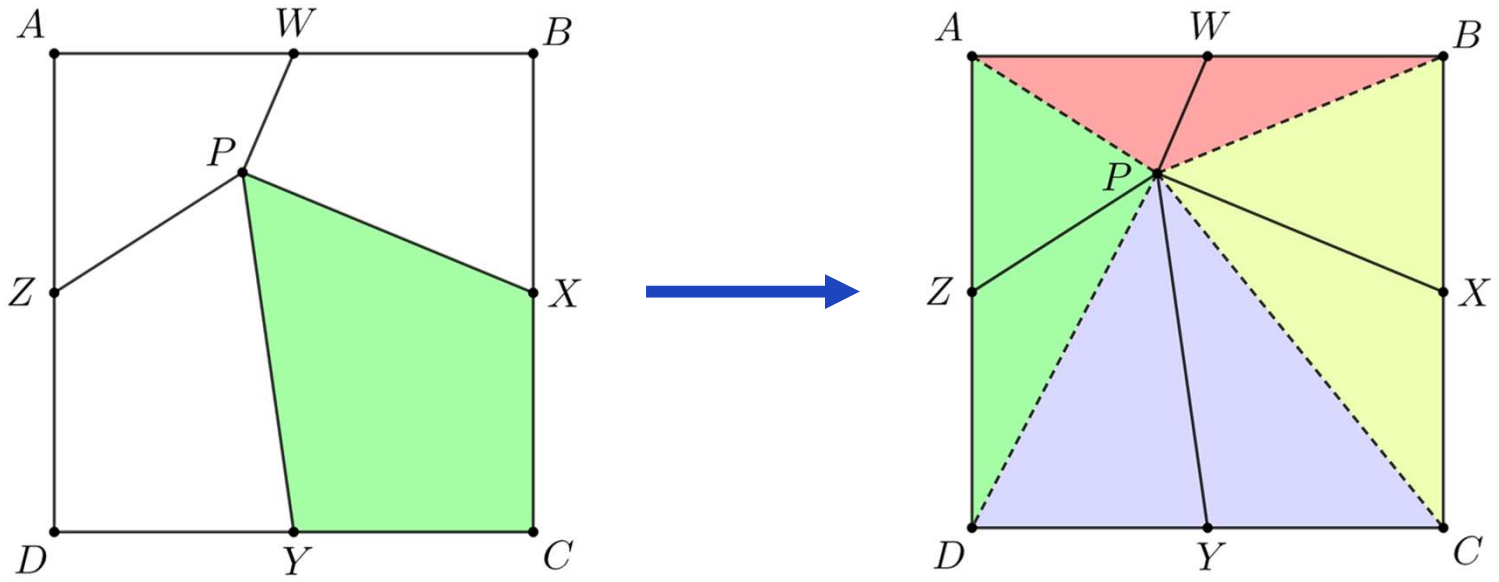
By same logic, area of CYD and EZF are also 6. So, area of ABCDEF is $4 \times 3 + 3 \times 6 = 39$.

Q8. A Windmill inside a Square

In the figure, ABCD is a square and W, X, Y, Z are midpoints of the corresponding sides. Let areas of AWPZ, BWPX and DYPZ be 10, 15 and 18 respectively. What is the area of CXPY?



Q8. A Windmill inside a Square



Triangles of the same colour have equal area.

Green + Blue + Red + Yellow is $15 + 18 = 33$.

Green + Red is 10.

Therefore, Blue + Yellow is $33 - 10 = 23$.

That's it for today.

I guess we both earned our rest!

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