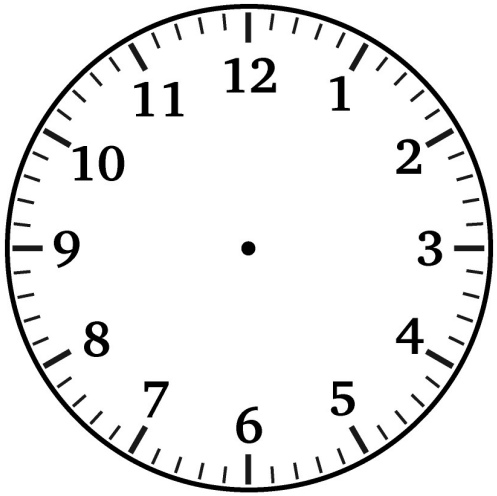
# Let's begin at 07:03 PM

Try this problem in the mean time. :)

Can you draw a line through the clock given the picture so that

- there are six numbers on each side of the line,
- sum of the numbers on each side are equal.




The answer will be posted when the notes are uploaded to the google classroom.

# Answer to the Intro Problem

The idea is to pair up the numbers as follows:

12 and 1

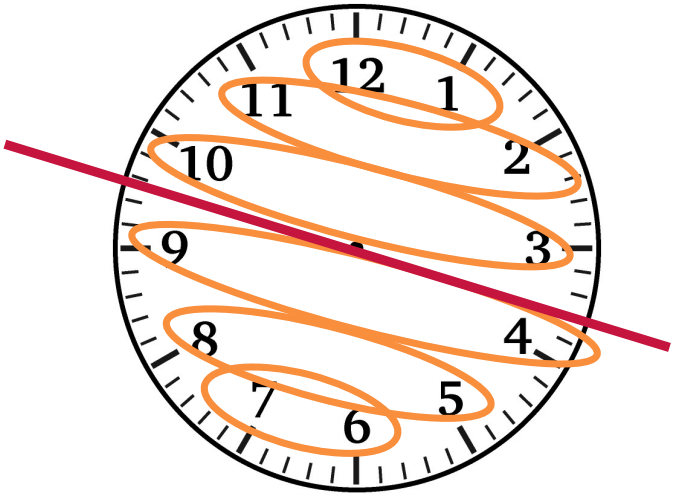
11 and 2

10 and 3

9 and 4

8 and 5

7 and 6




Sum of each pair is equal (to 13). Thus, if we keep three pairs each on each side of the line, the sums will be equal.

# Some Housekeeping

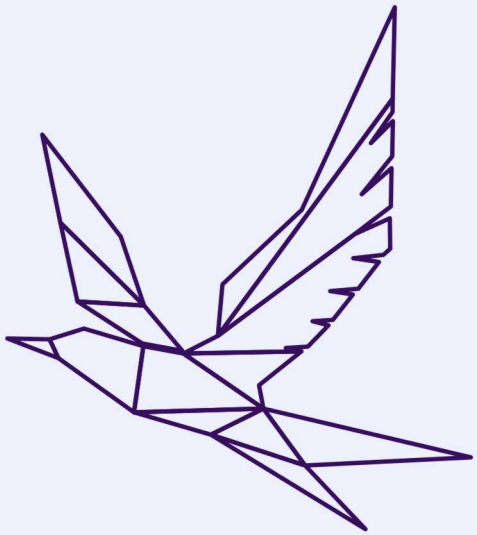
- Here are my details:
  - Name: Hein Thant Aung
  - Contact: heinta2002@gmail.com (or Facebook, link in outline)
  - Was a contestant of IMO 2017.
  - Currently a math undergraduate at Chinese University of Hong Kong.

- Everything about this course will be done in google classroom. So, please make sure to join!

- We will have a mini-assignment every week, and a test at the end.

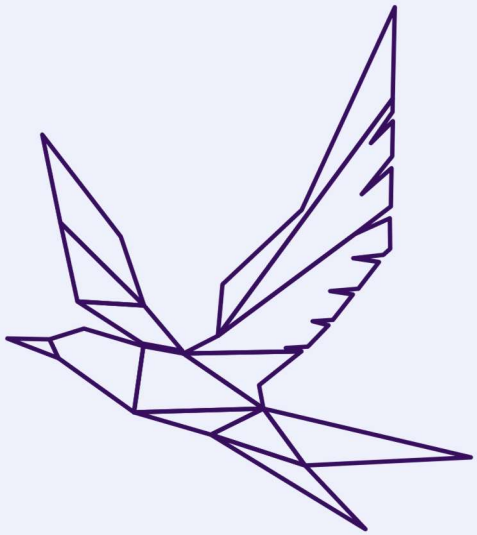
# Lecture – 01

# Basic Counting Principles



# Section – I

## The Multiplication Principle

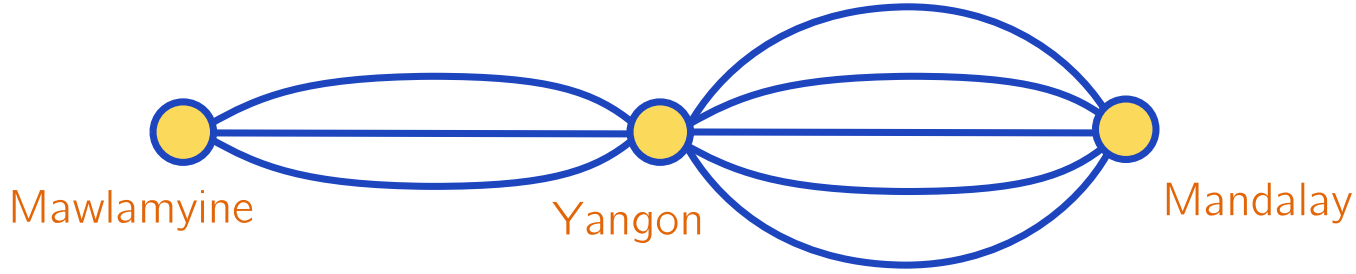


# Let's Start with a Question

Min Min wants to travel from Mawlamyine to Yangon, then from Yangon to Mandalay.

- There are 3 ways to travel from Mawlamyine to Yangon,
- There are 5 ways to travel from Yangon to Mandalay.

In how many ways can he plan his trip?

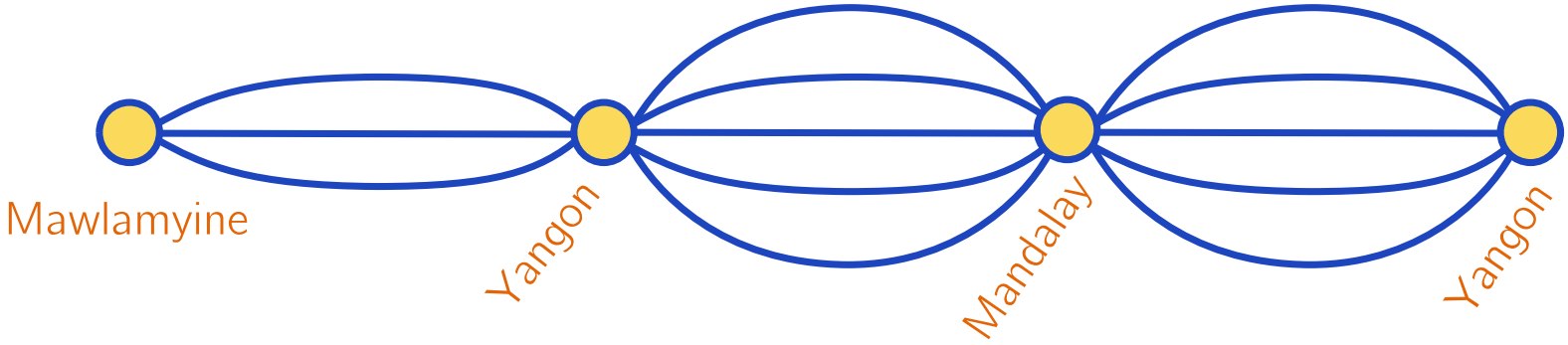


# Let's Make it Harder

Min Min wants to travel from Mawlamyine to Yangon, then from Yangon to Mandalay, then from Mandalay back to Yangon.

- There are 3 ways to travel from Mawlamyine to Yangon,
- There are 5 ways to travel between Mandalay and Yangon.

In how many ways can he plan his trip?

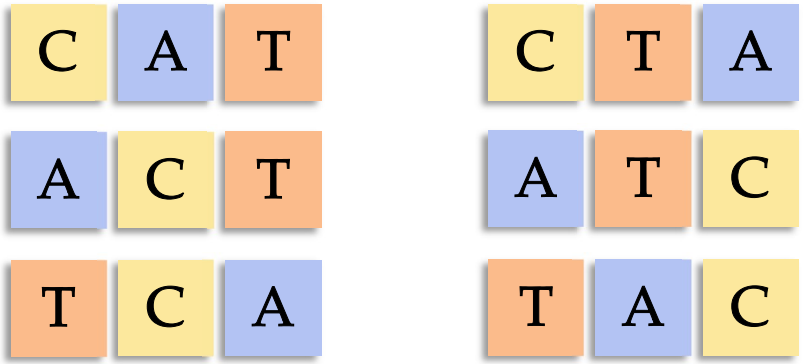


# Q1: Shuffling Letters

Min Min has three cards. On one card, letter C is written. On one card, letter A is written. On the last card, letter T is written.

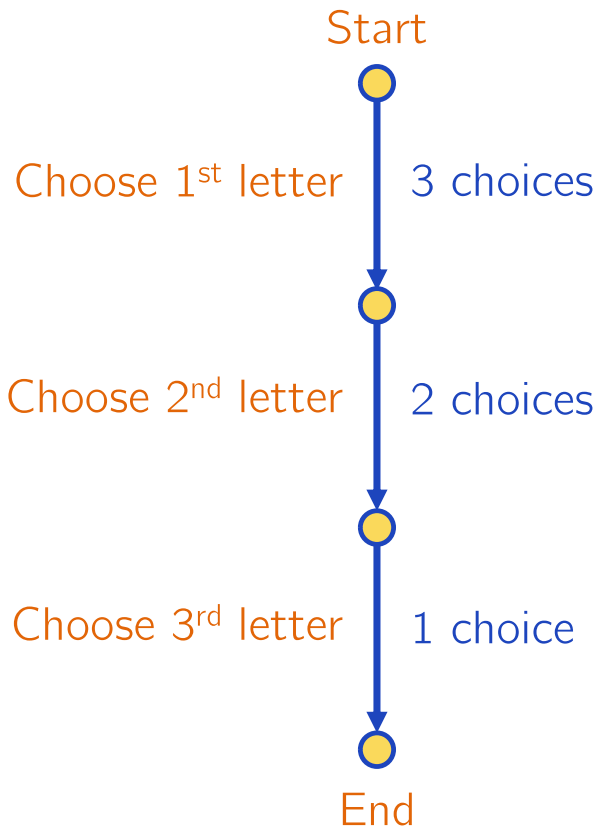
He wants to make a 3-letter word (possibly meaningless in English).

How many words can he make? Answer: 6




# Q1: Shuffling Letters

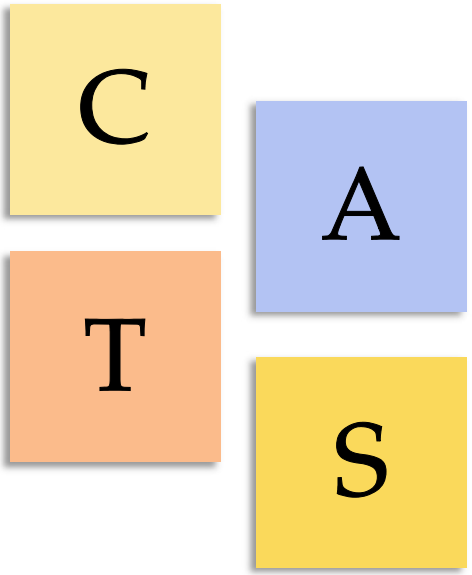
This problem can also be thought of as travelling problem!




# Q1': Shuffling More Letters

Now, Min Min got one more card on which the letter S is written.

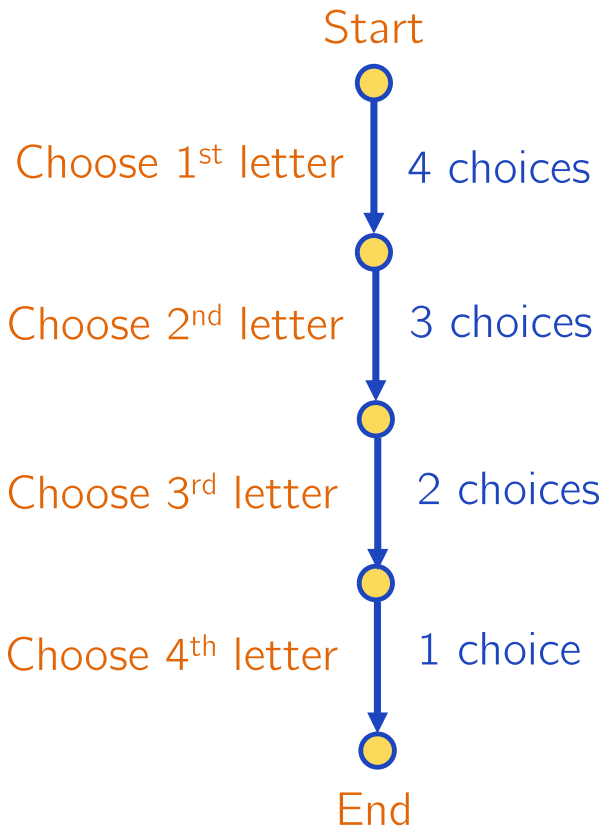
How many 4-letter words can he make? Answer: 24

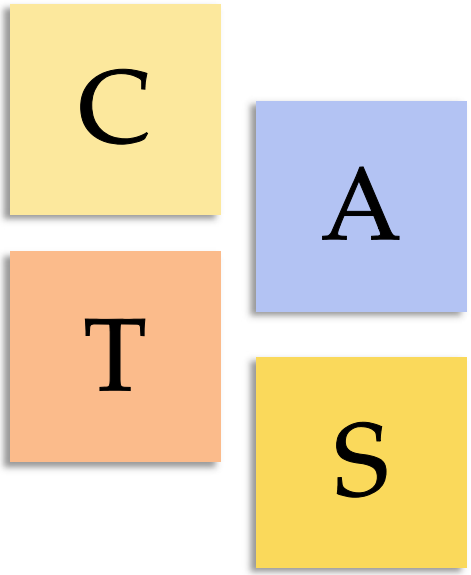



| | | | |
|---|---|---|---|
| C A T S | A C T S | T A C S | S A T C |
| C A S T | A C S T | T A S C | S A C T |
| C T A S | A T C S | T C A S | S T A C |
| C T S A | A T S C | T C S A | S T C A |
| C S A T | A S C T | T S A C | S C A T |
| C S T A | A S T C | T S C A | S C T A |

# Q1': Shuffling More Letters

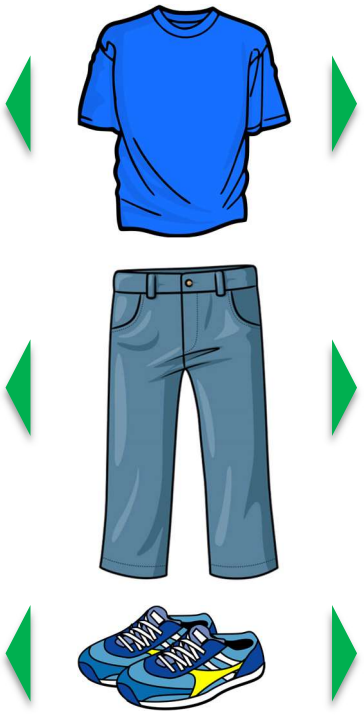
This problem can also be thought of as travelling problem!



# Q2: Choosing your Outfit

Aung Ko has 8 shirts, 4 pairs of pants and 5 pairs of shoes. How many different outfits can he create? **Answer: 160** ← $8 \times 4 \times 5$

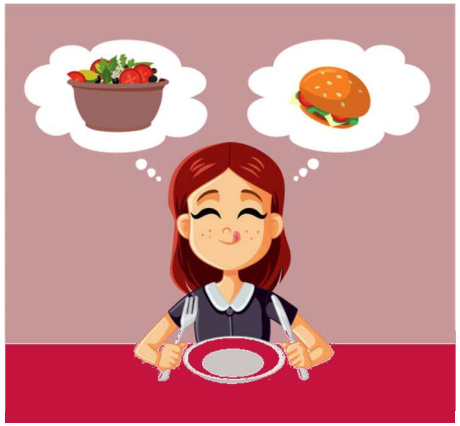



# Q3: Ordering in a Restaurant

A restaurant offers 6 appetizers, 4 main courses and 4 desserts. Sandy wants to order the following sequence of food:

- one dessert, one appetizer, one main course, and finally one dessert that is different from the first one.

In how many different ways can she arrange her menu?



Answer: 288 $\leftarrow 4 \times 6 \times 4 \times 3$

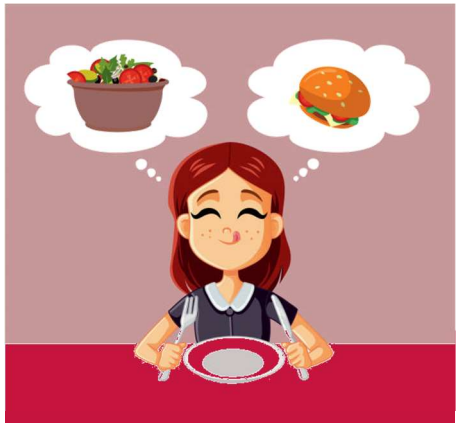
| 1st Dessert | Appetizer | Main Course | 2nd Dessert |
|---|---|---|---|
| 4 choices | 6 choices | 4 choices | 3 choices |

# Q4: Ordering in a Restaurant (again)

A restaurant offers 6 appetizers, 4 main courses and 4 desserts. Sandy wants to order the following sequence of food:

- one dessert, one appetizer, one main course, and finally one dessert that is different from the first one.

But, she might skip the appetizer if she feels quite full. How many ways can she arrange her menu? Answer: 336 $\leftarrow 4 \times 7 \times 4 \times 3$



| 1st Dessert | Appetizer | Main Course | 2nd Dessert |
|---|---|---|---|
| 4 choices | 7 choices | 4 choices | 3 choices |

6 original choices + choice to skip

# Q5: Crafting a 3-digit Number

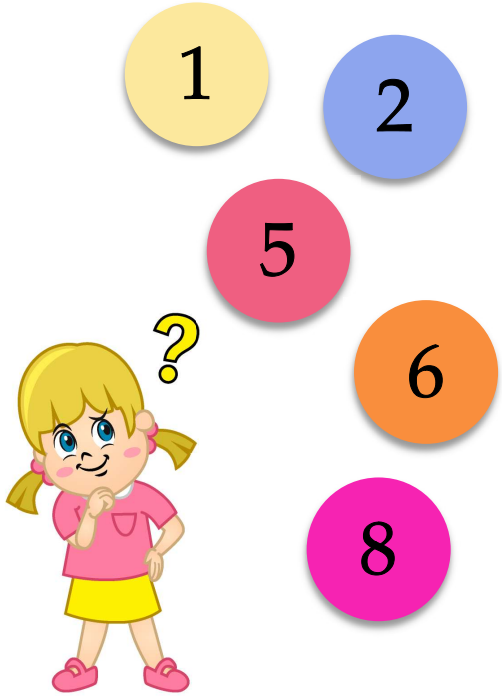
Hnin Pwint wants to create 3-digit numbers using the digits 1, 2, 5, 6 and 8 at most once each.

a) How many different numbers can she create?

Answer: 60 ← $5 \times 4 \times 3$

b) How many even numbers can she create?

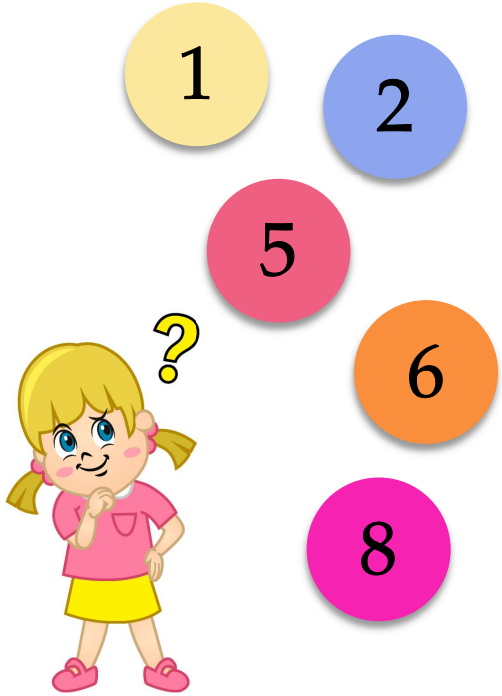
Answer: 36 ← $3 \times 4 \times 3$




| Last digit | Hundred's digit | Ten's digit |
|---|---|---|
| 3 choices | 4 choices | 3 choices |

# Order of Decisions Matters

b) How many even numbers can she create?




How NOT to do this question

| Hundred's digit | Ten's digit | Last digit |
|---|---|---|
| 5 choices | 4 choices | ??? choices |

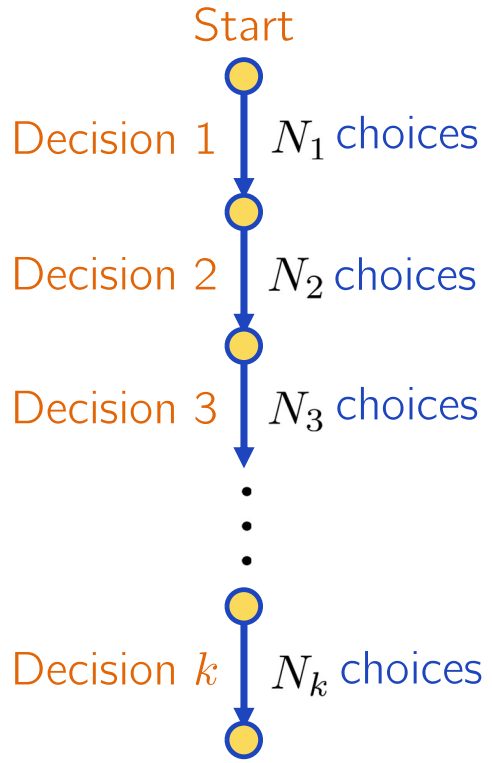
Number of choices depend on previous decisions.

# Multiplication Principle

We want to make a sequence of $k$ decisions.

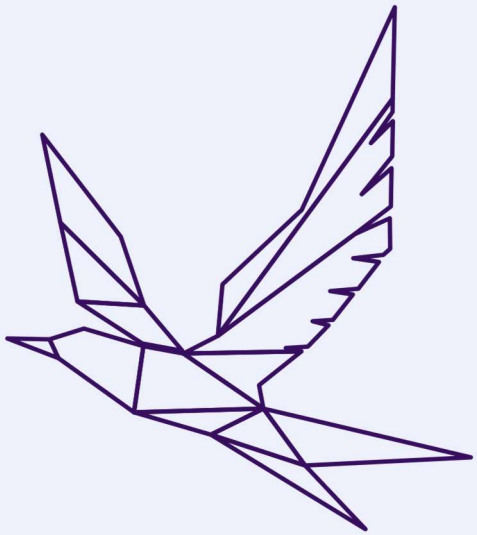
- First decision can be done in $N_1$ different ways.
- Second decision can be done in $N_2$ different ways.
- Third decision can be done in $N_3$ different ways.
- ...
- The $k$-th decision can be done in $N_k$ different ways.

Suppose that the previous decisions do not affect the number of choices for the next decisions. Then, number of ways to carry out the decisions in that order is equal to $N_1 \times N_2 \times N_3 \times \cdots \times N_k$.



# Section – II

## The Addition Principle

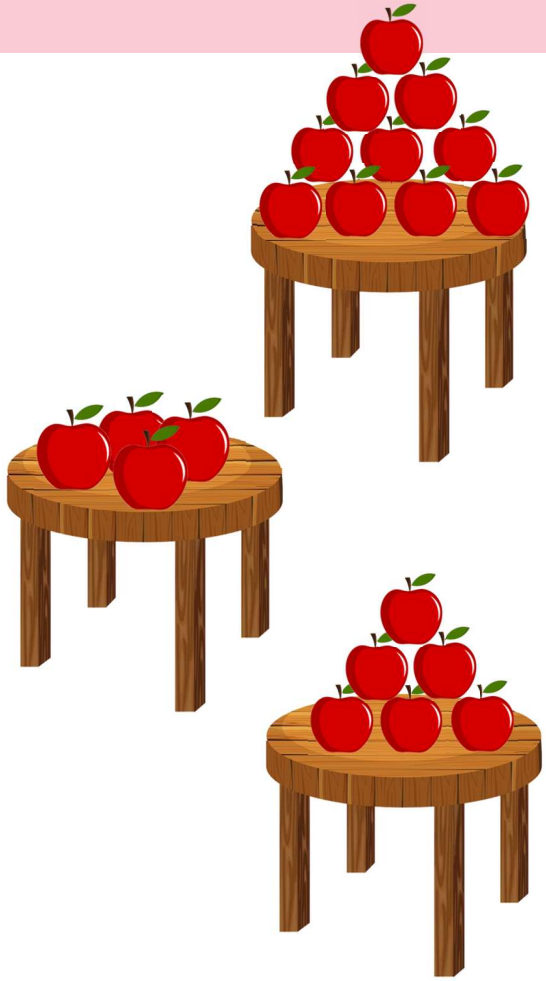


# Q6: Kindergarten Math

There are three tables in a room. There are 4 apples on the first table, 6 apples on the second table and 10 apples on the third. How many apples are there on all three tables combined?

Answer: 20 ← $4 + 6 + 10$

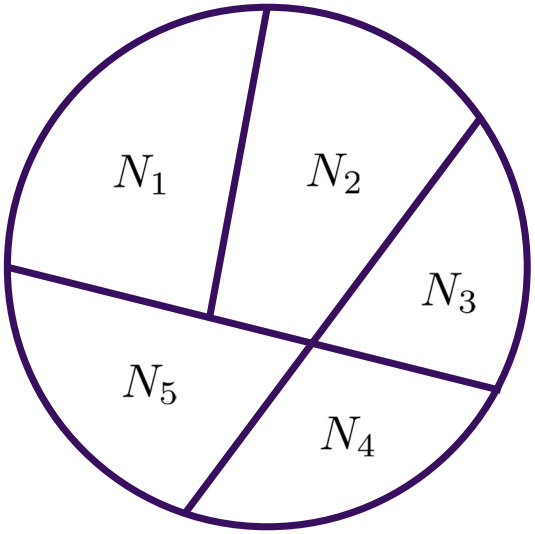
Well, this is probably too easy for you!



# Addition Principle

Suppose that the objects we want to count can be separated into $k$ non-intersecting groups. Suppose that

- there are $N_1$ objects in the 1st group,
- there are $N_2$ objects in the 2nd group,
- there are $N_3$ objects in the 3rd group.
- ...
- there are $N_k$ objects in the $k$-th group.




Then, the total number of objects is equal to $N_1 + N_2 + N_3 + \cdots + N_k$ .

# Q7: Arranging Coins

Hein Min has 4 different blue coins and 6 different red coins. He wants to arrange five of his coins in a row so that the colours alternate between red and blue. In how many ways can he do this?

Answer: 2160 ← $1440 + 720$

Solution

Split the possibilities into two groups:

|  |  |
|---|---|
| $6 \times 4 \times 5 \times 3 \times 4$ | $4 \times 6 \times 3 \times 5 \times 2$ |
| ways | ways |

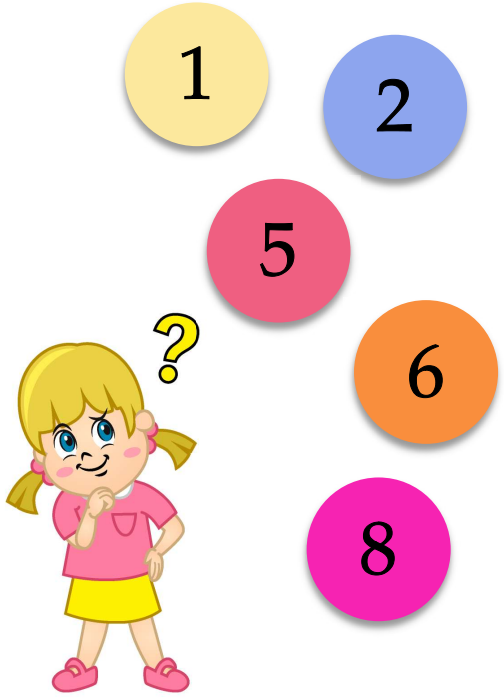
# Q8: Crafting a 3-digit Number (again)

Hnin Pwint wants to create 3-digit numbers using the digits 1, 2, 5, 6 and 8 at most once each. How many even numbers can she create that does not exceed 600?

Difficulty: Choices for hundred's and last digits are clashing!

| Hundred's digit | Last digit | Ten's digit |
|---|---|---|
| 3 choices | ??? choices | 3 choices |

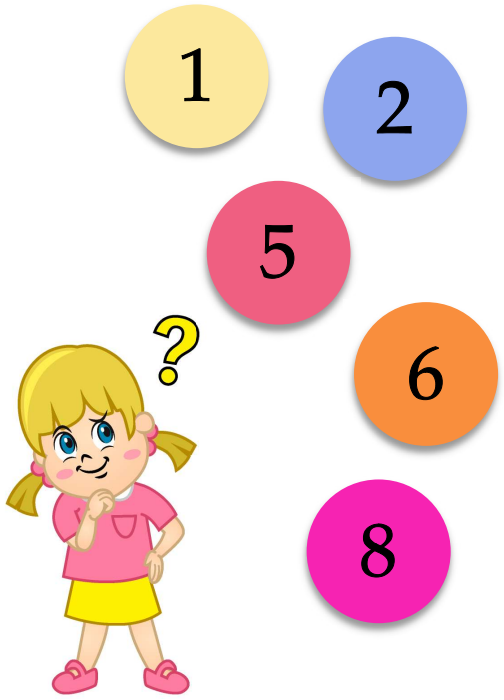
| Last digit | Hundred's digit | Ten's digit |
|---|---|---|
| 3 choices | ??? choices | 3 choices |



# Q8: Crafting a 3-digit Number (again)

Hnin Pwint wants to create 3-digit numbers using the digits 1, 2, 5, 6 and 8 at most once each. How many even numbers can she create that does not exceed 600?

**Answer: 24** ← $6 + 18$




Solution

Hundred's digit is 2

$1 \times 2 \times 3$ ways

Hundred's digit is 1 or 5

$2 \times 3 \times 3$ ways

Well, I guess we both earned our rest.

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See you in the next lecture!